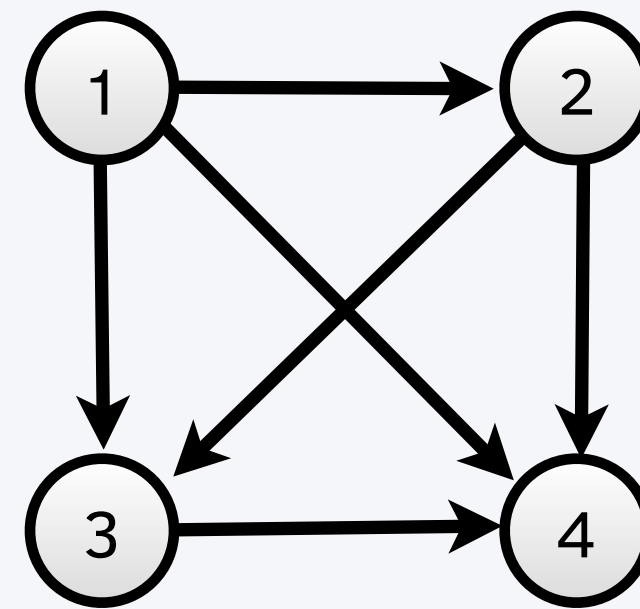


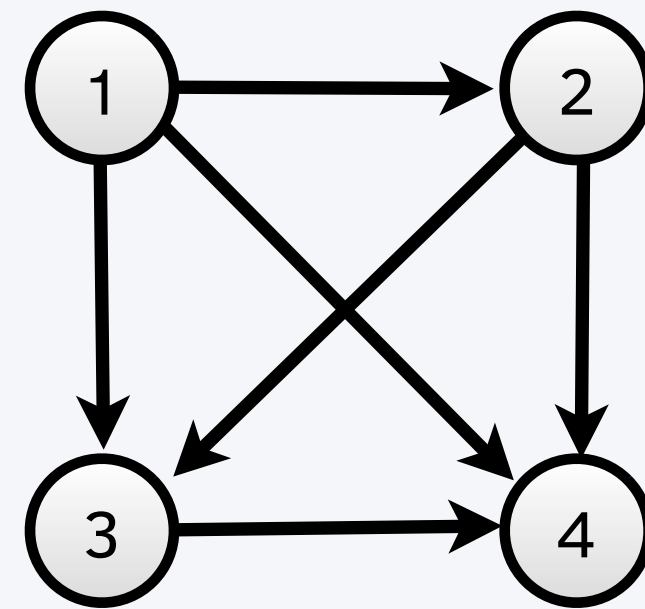
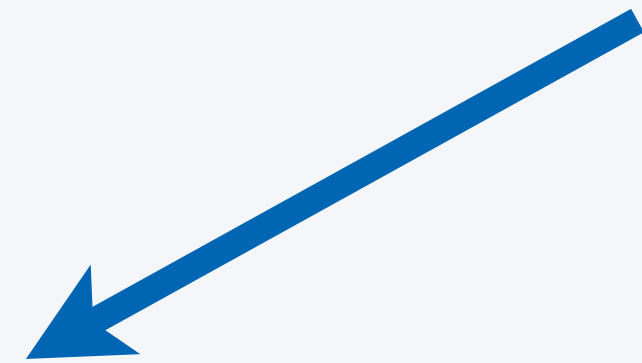
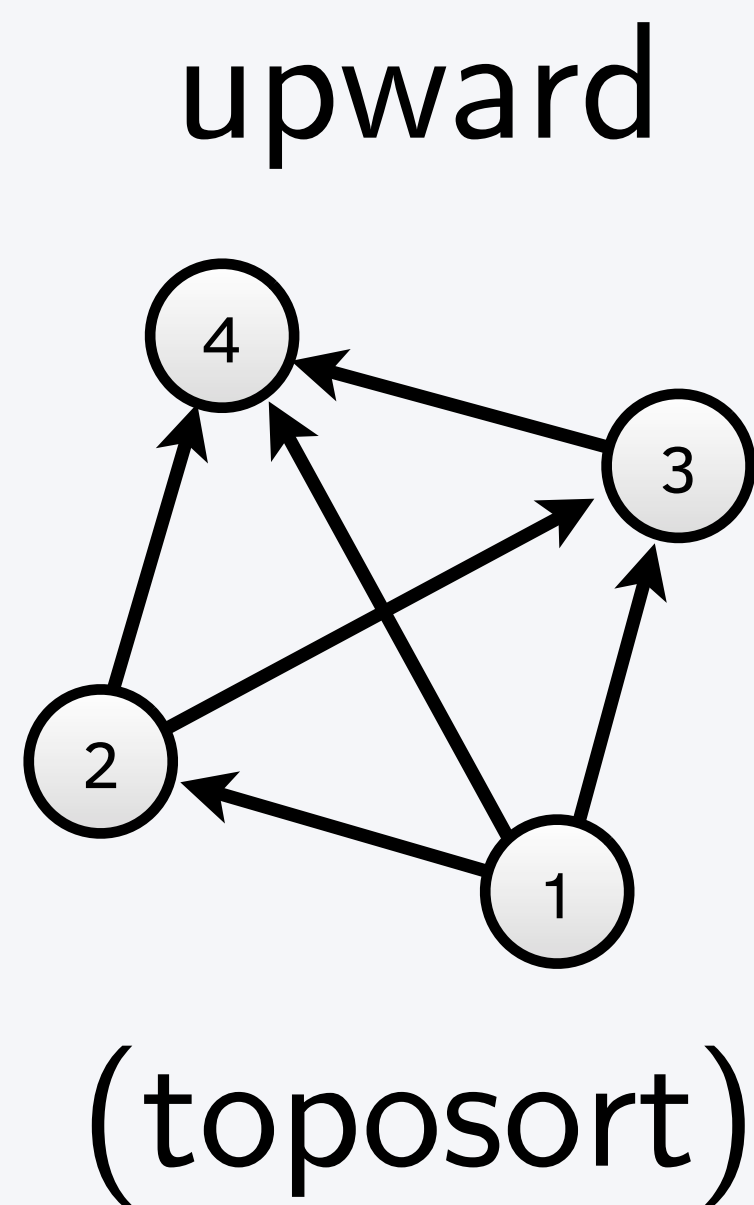
# Drawing Directed Graphs

ETH Zürich  
Daniel Graf



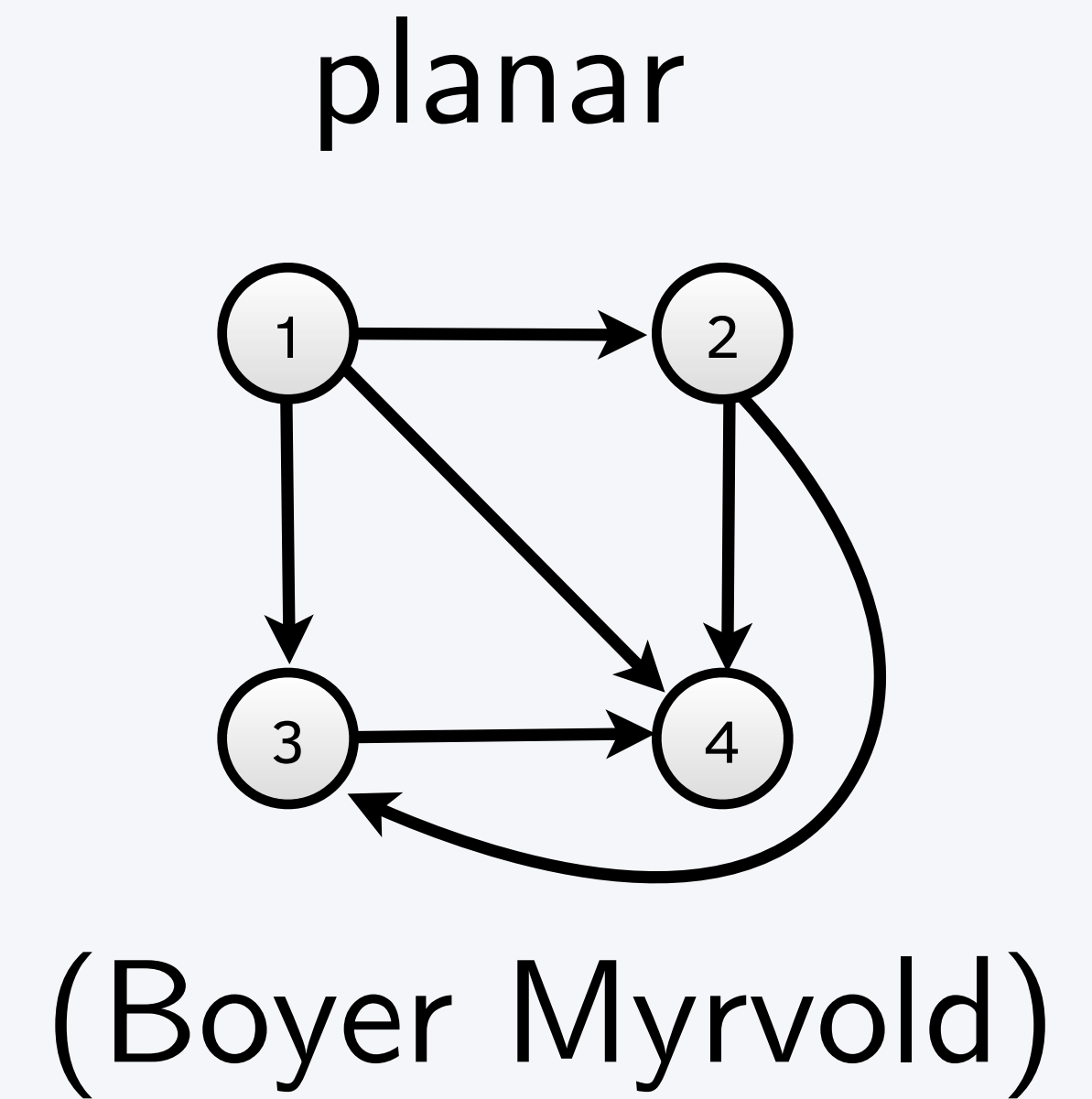
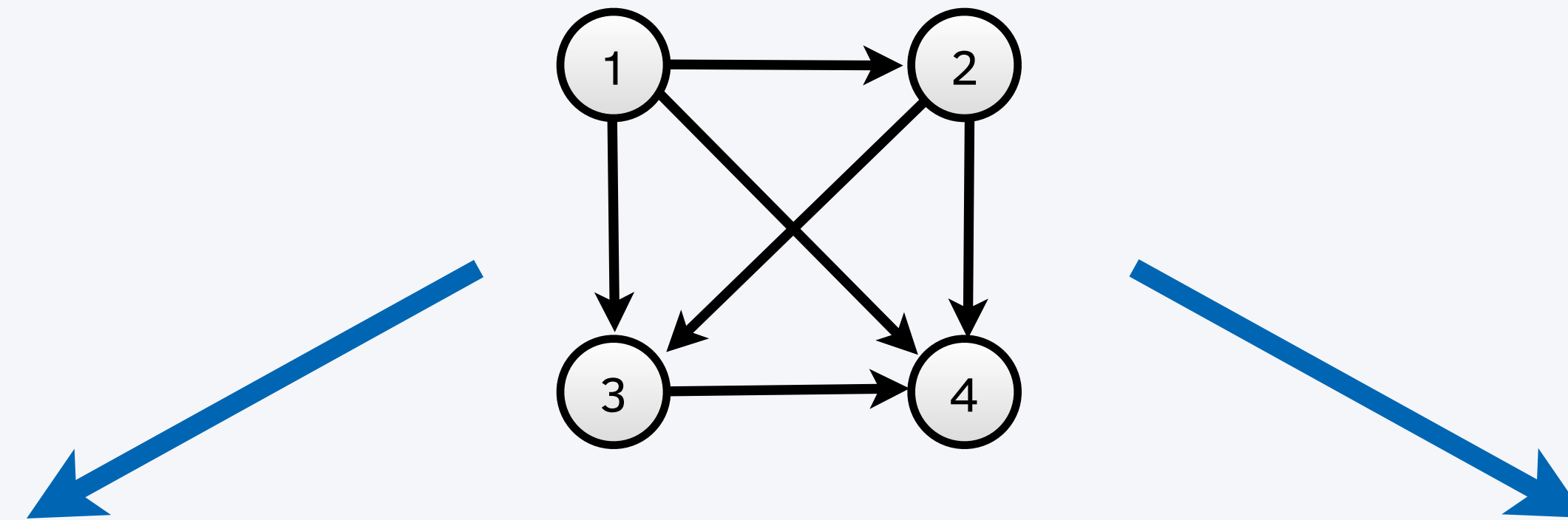
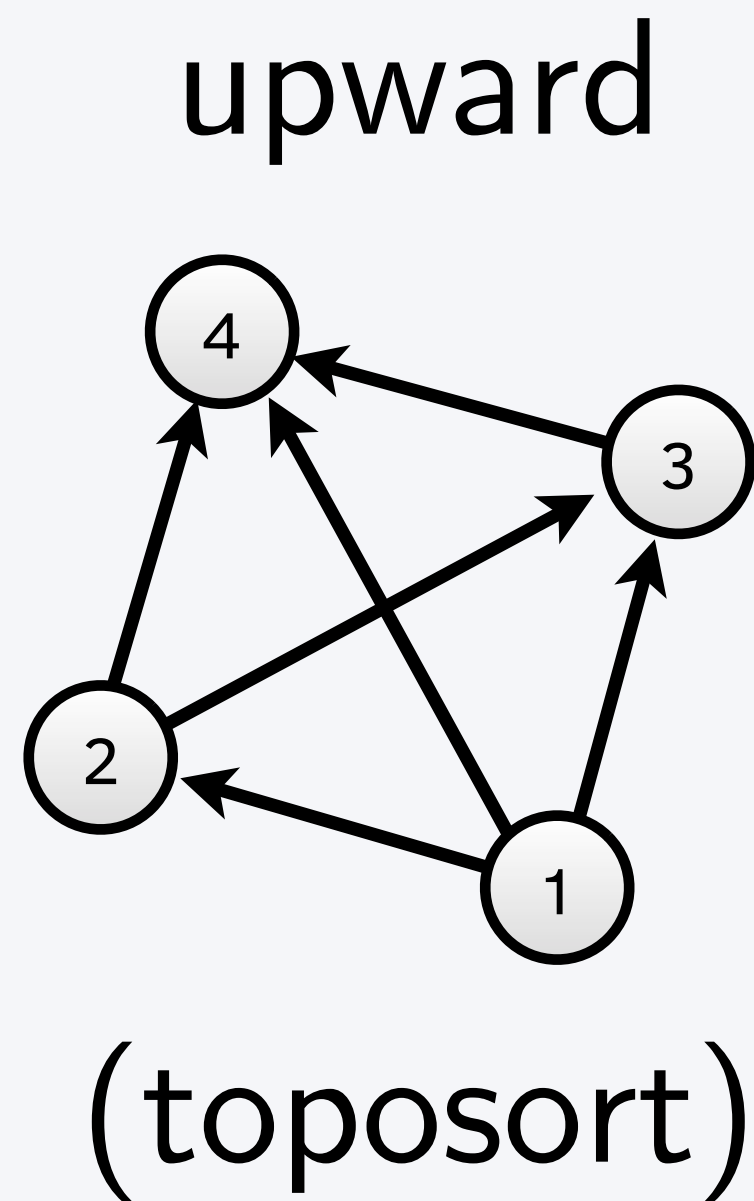
# Drawing Directed Graphs

ETH Zürich  
Daniel Graf



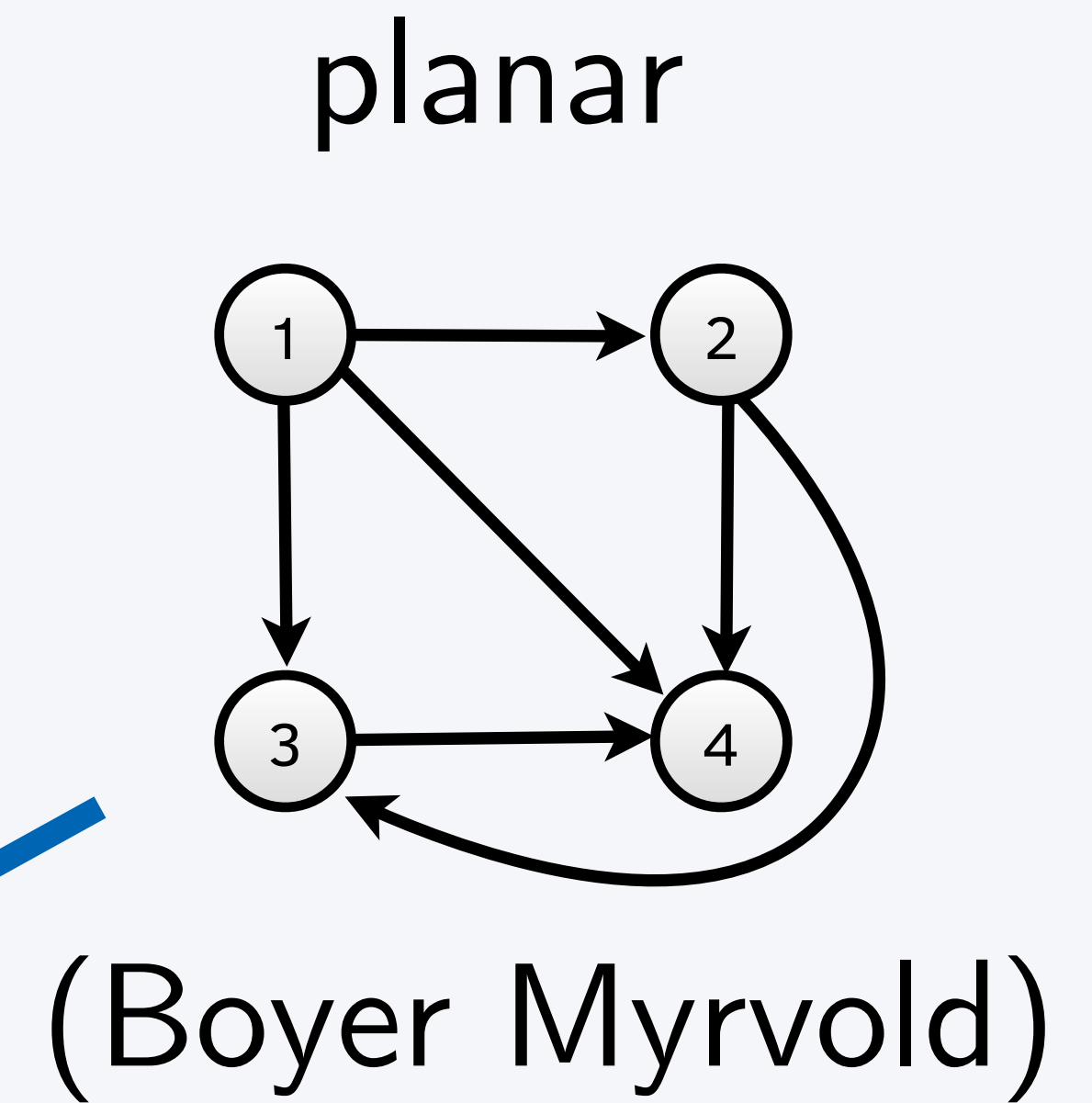
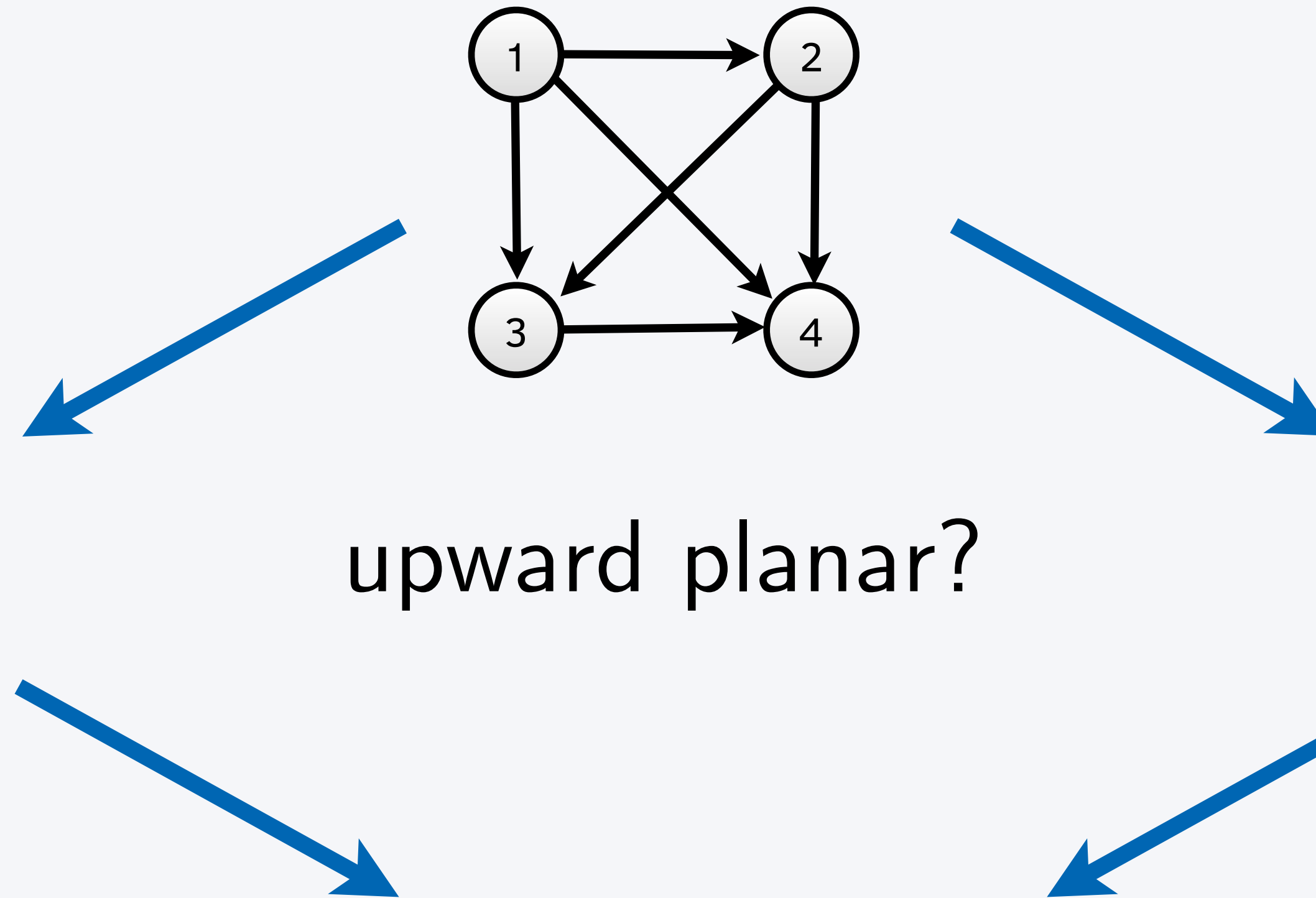
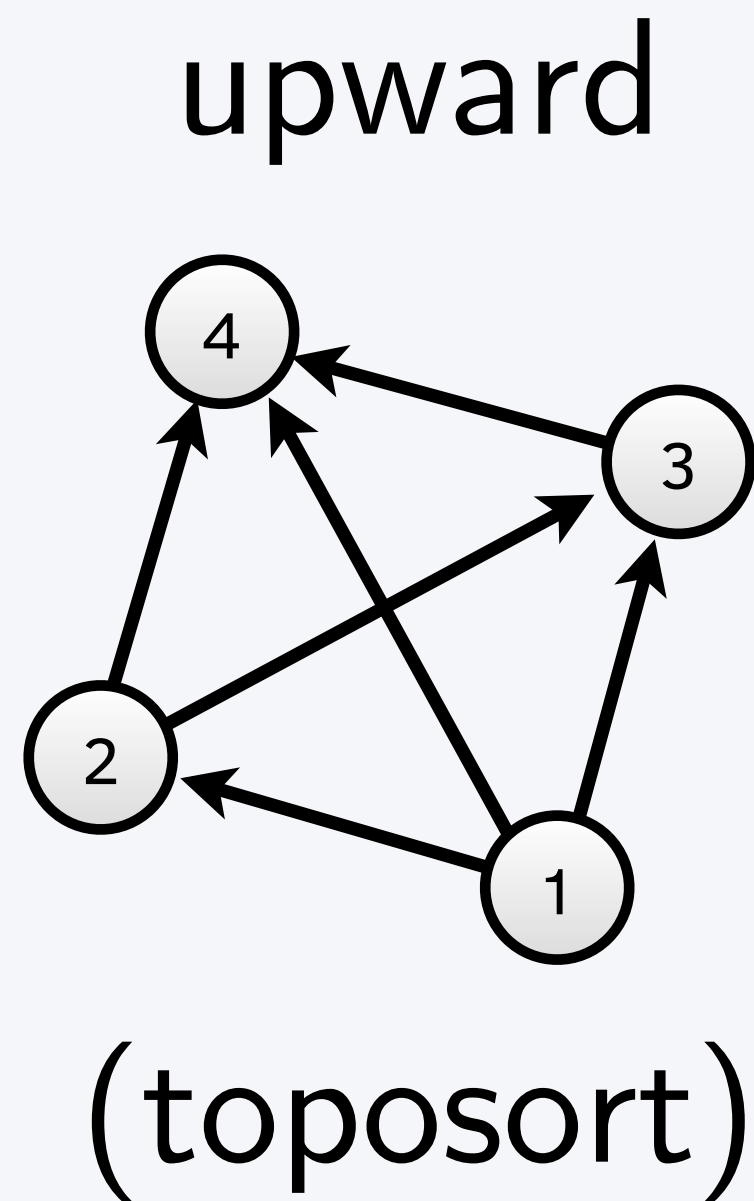
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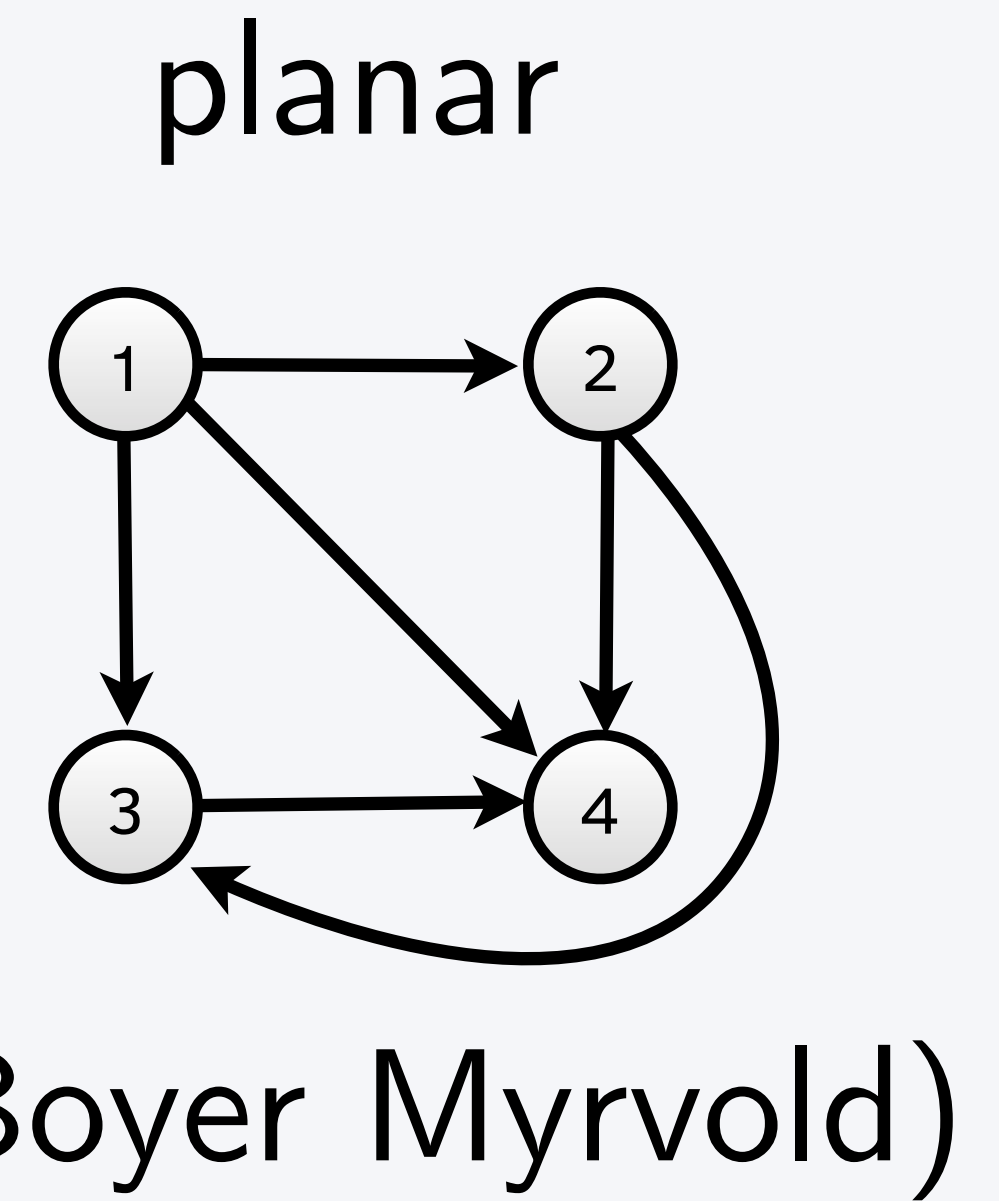
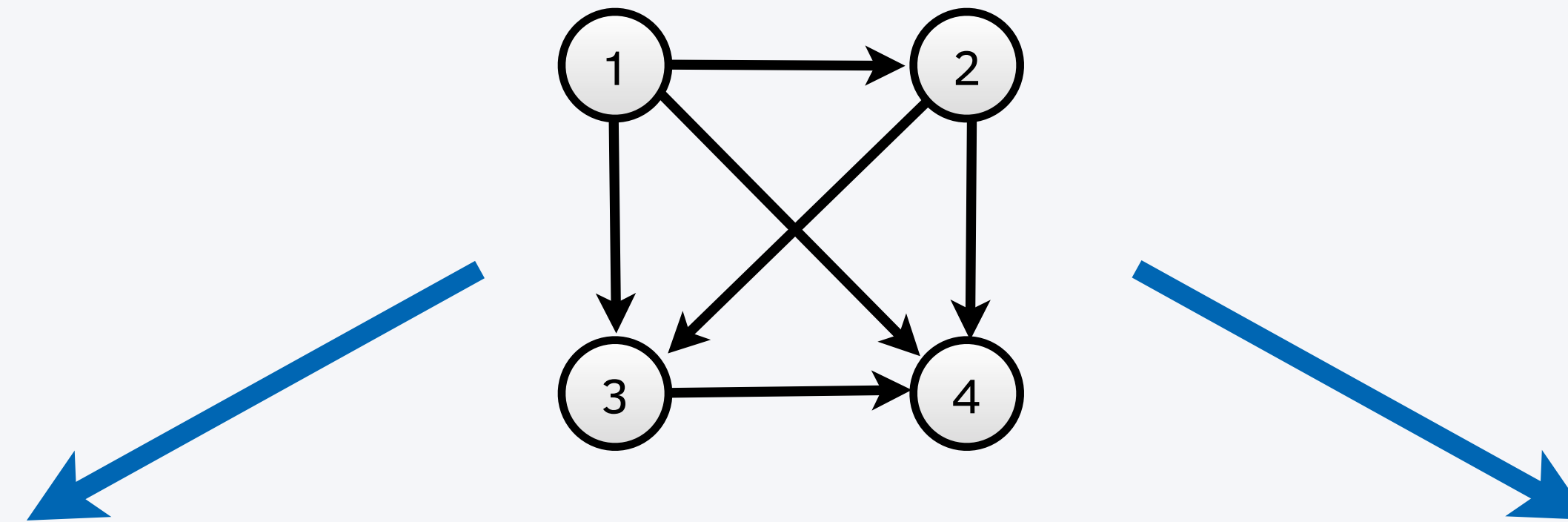
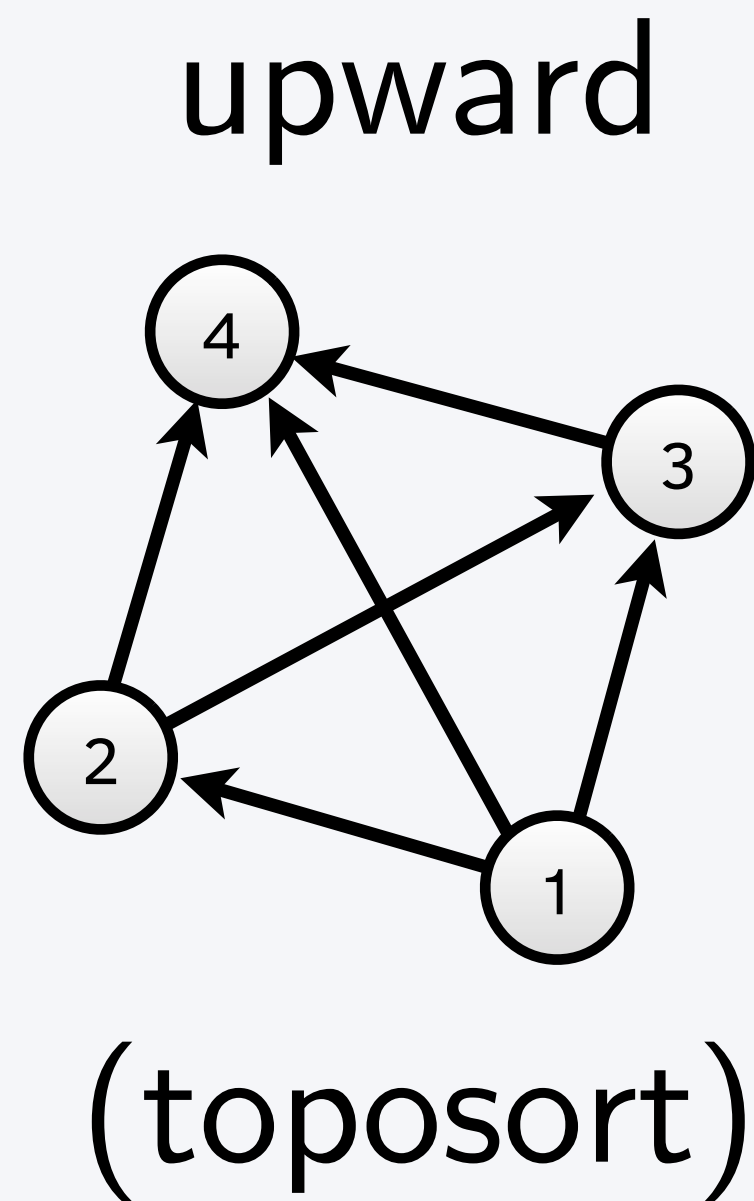
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Daniel Graf

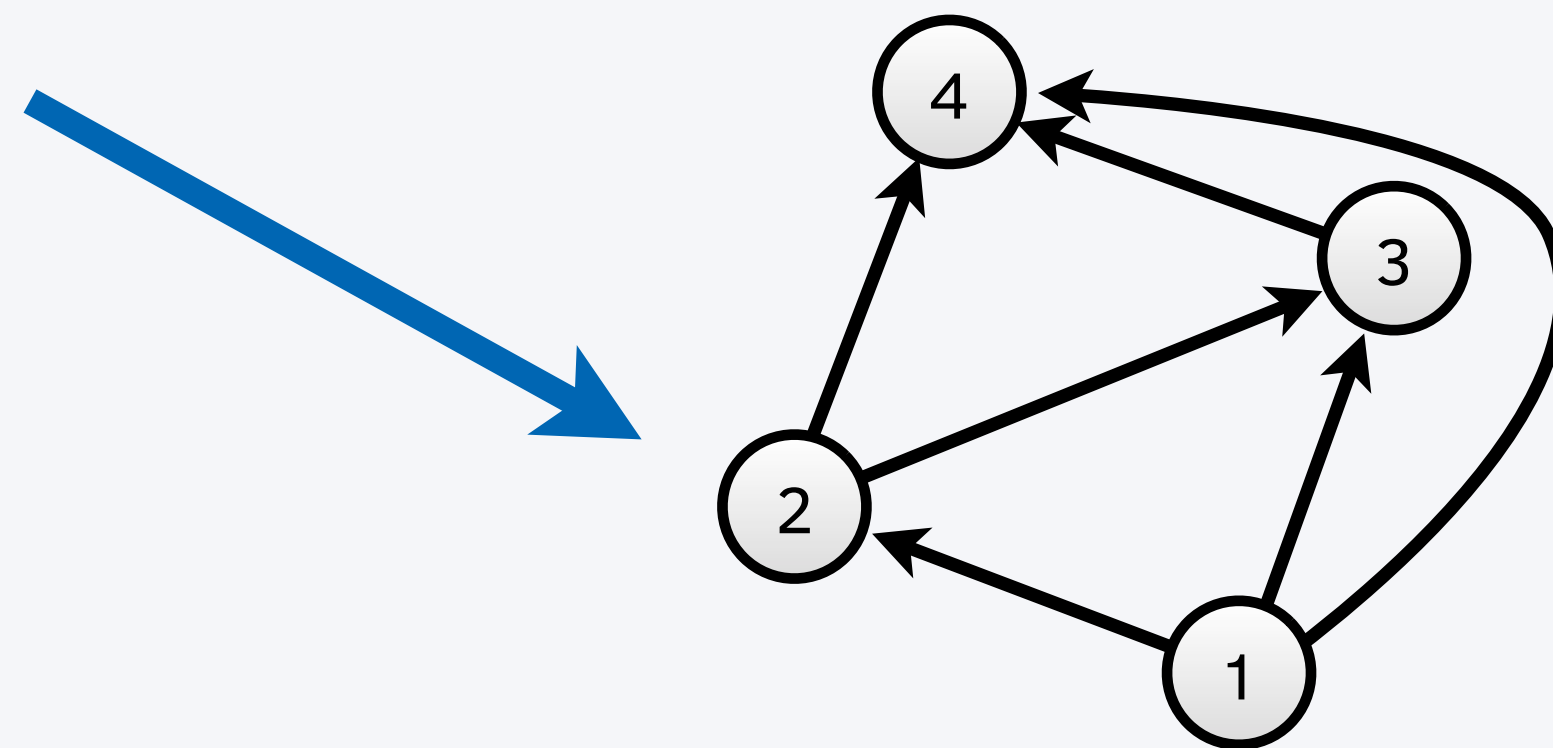


# Drawing Directed Graphs

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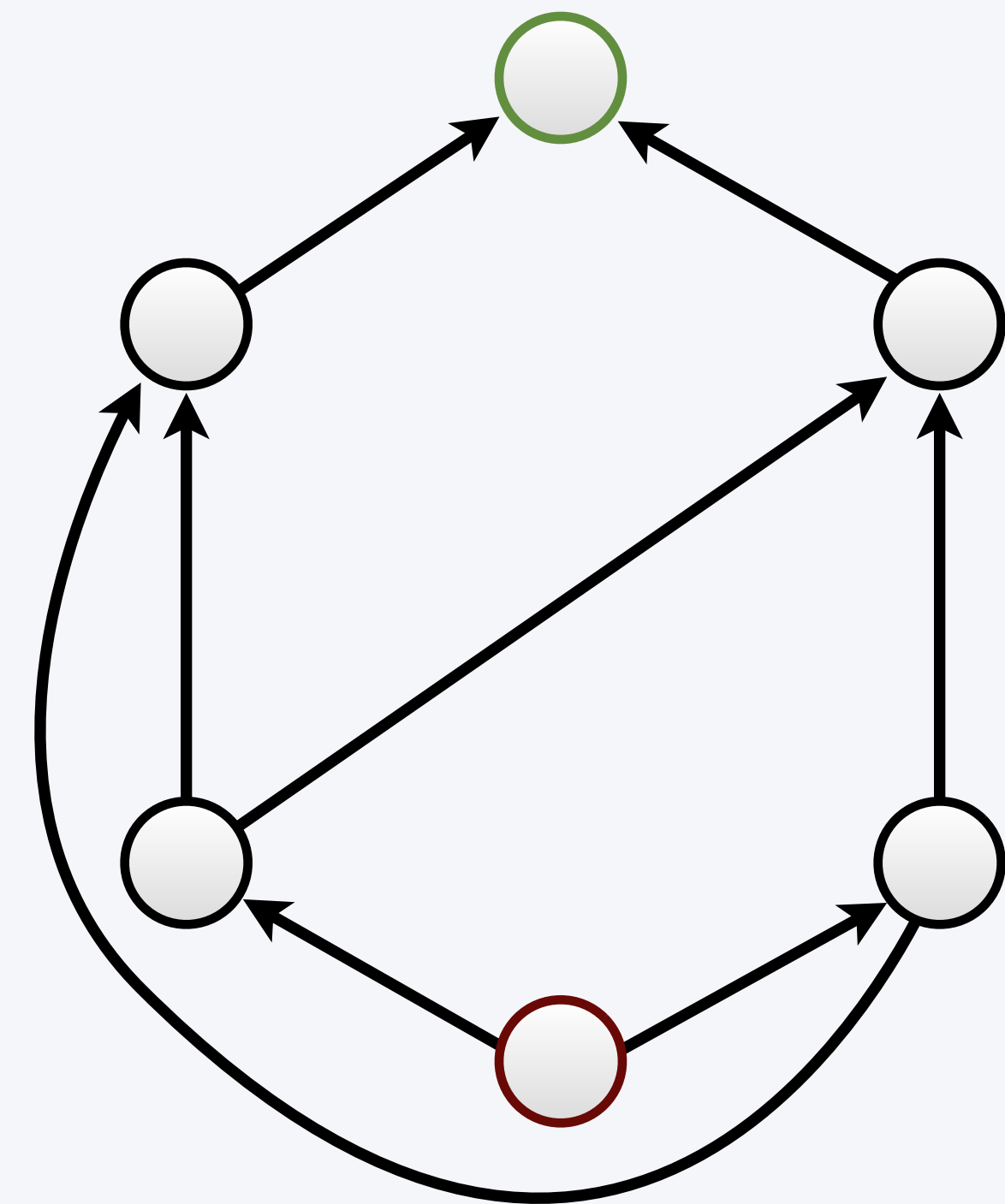
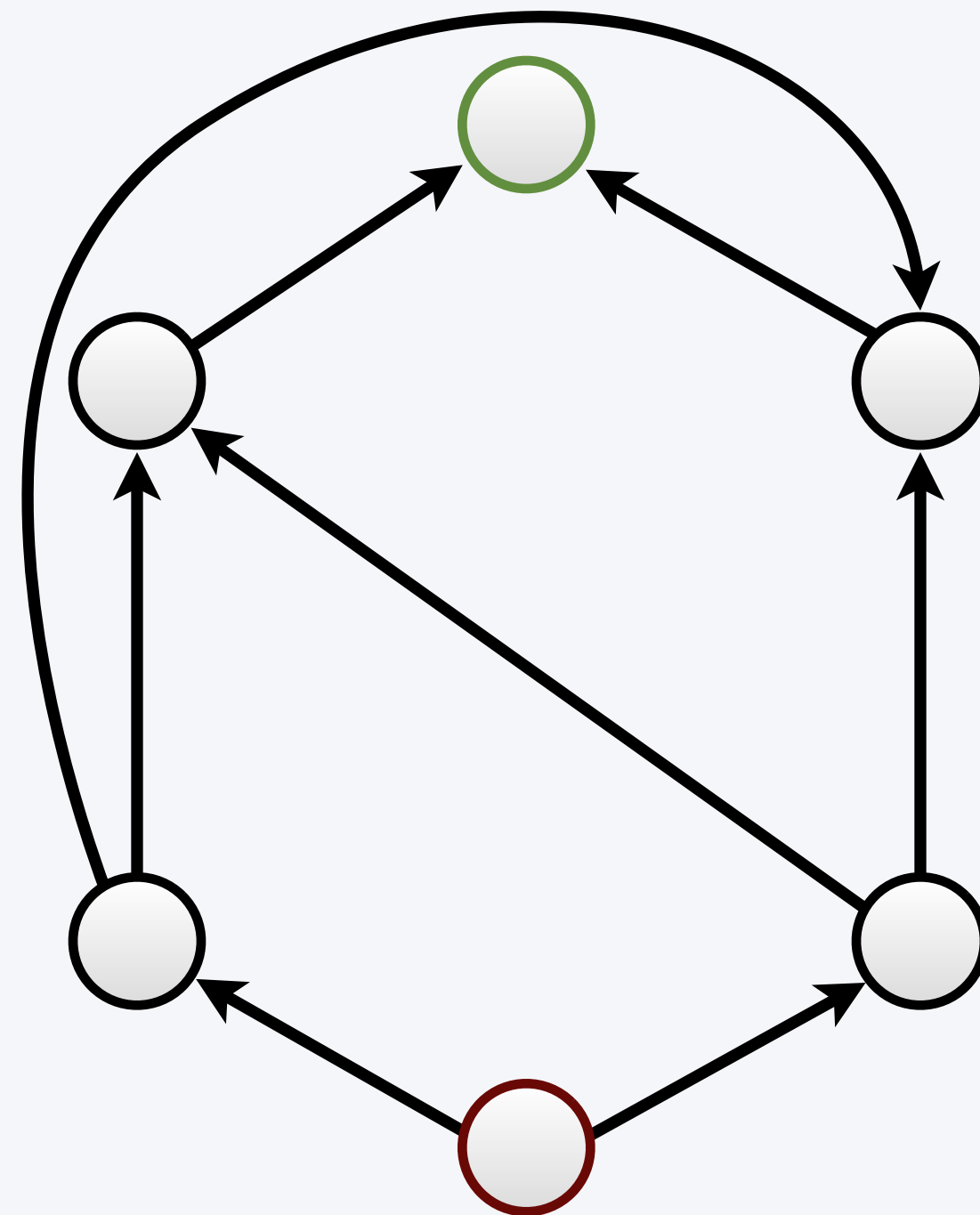
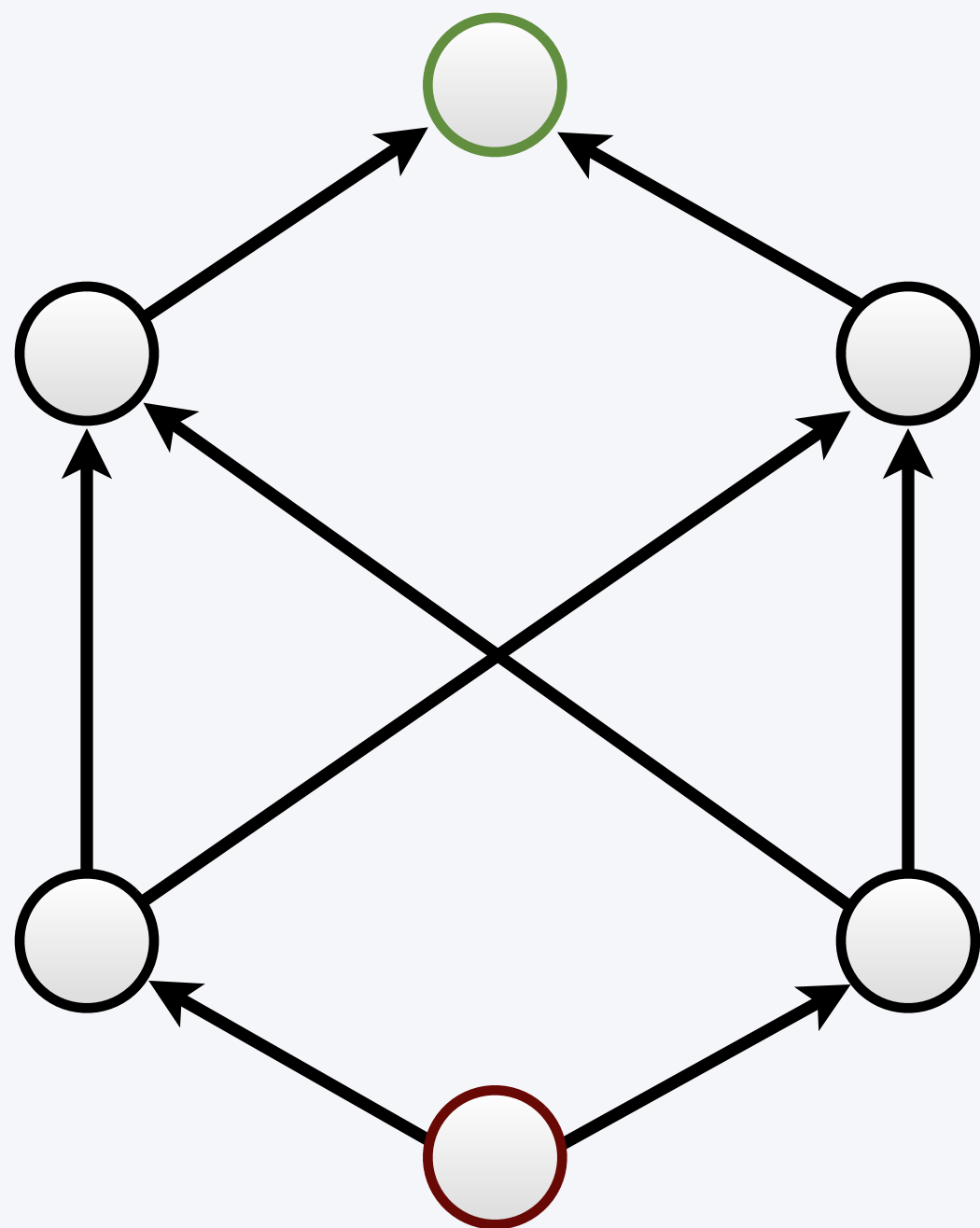
upward planar?



# Drawing Directed Graphs

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Acyclicity is not enough



# Checking Upward Planarity

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Daniel Graf

[GT95a] Ashim Garg and Roberto Tamassia. On the computational complexity of upward and rectilinear planarity testing. In *Graph drawing*, pages 286–297. Springer, 1995.

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# Checking Upward Planarity

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*NP*-hard in general [GT95a]

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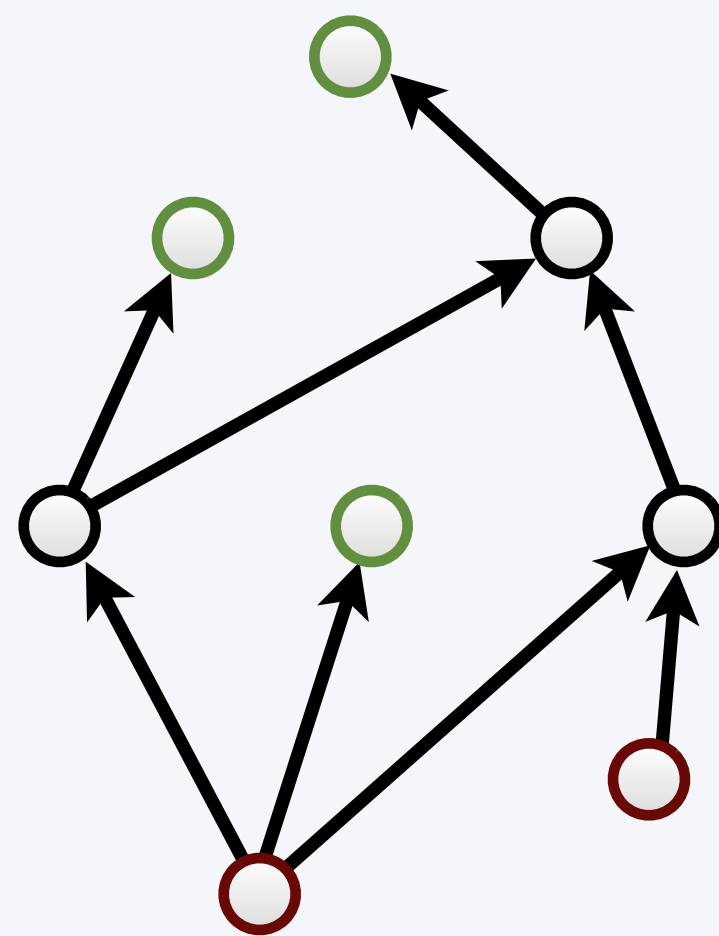
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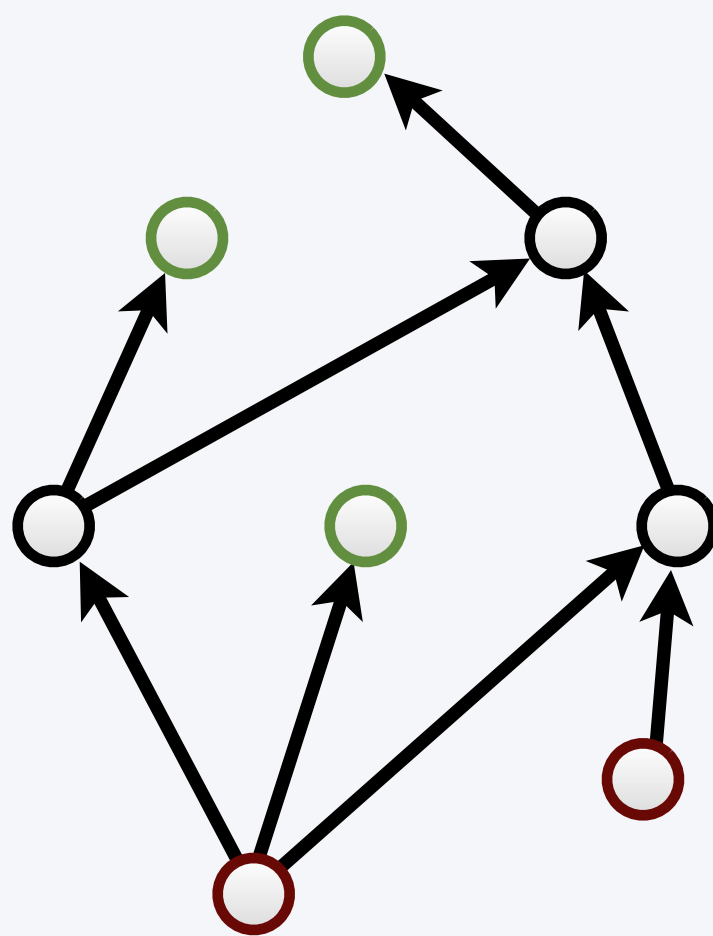
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ETH Zürich  
Daniel Graf

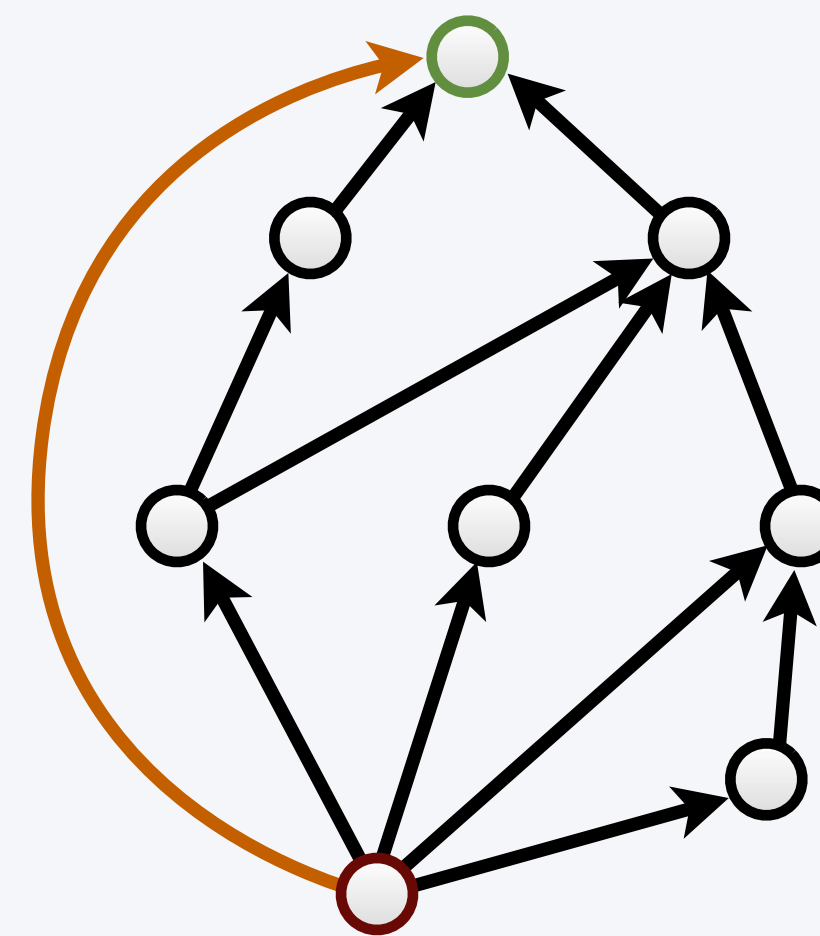
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*s-t*-planar



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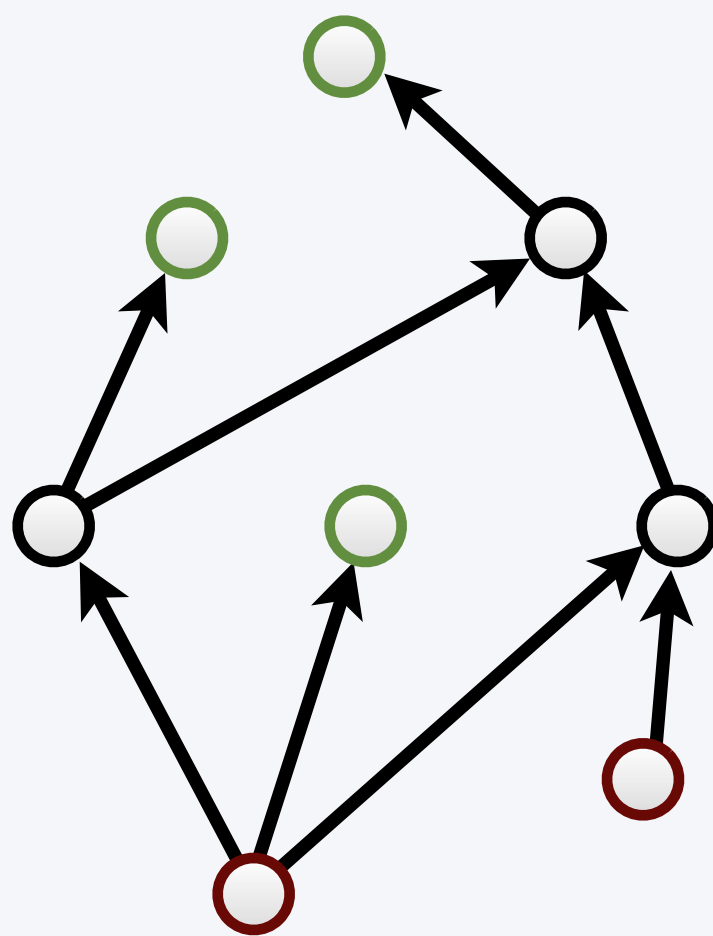
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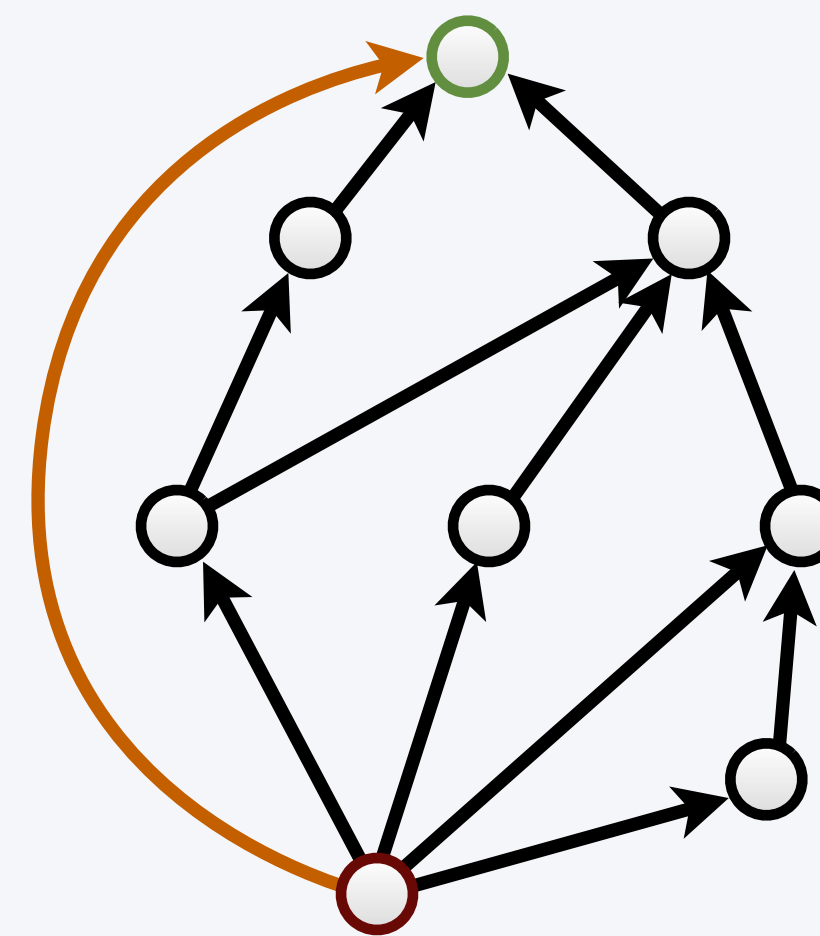
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is spanning  
subgraph of

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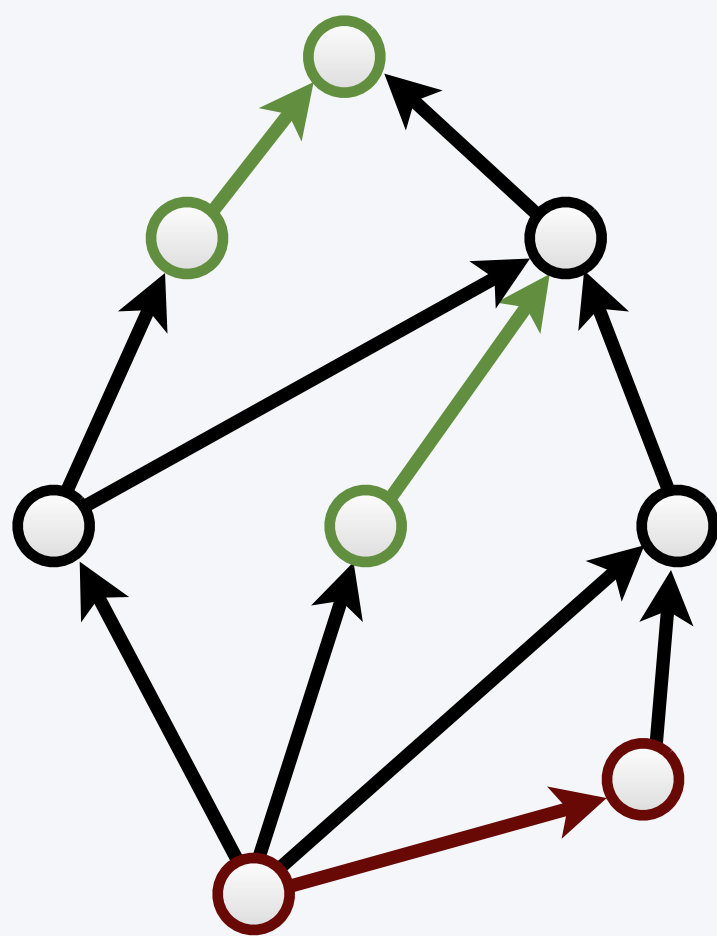
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ETH Zürich  
Daniel Graf

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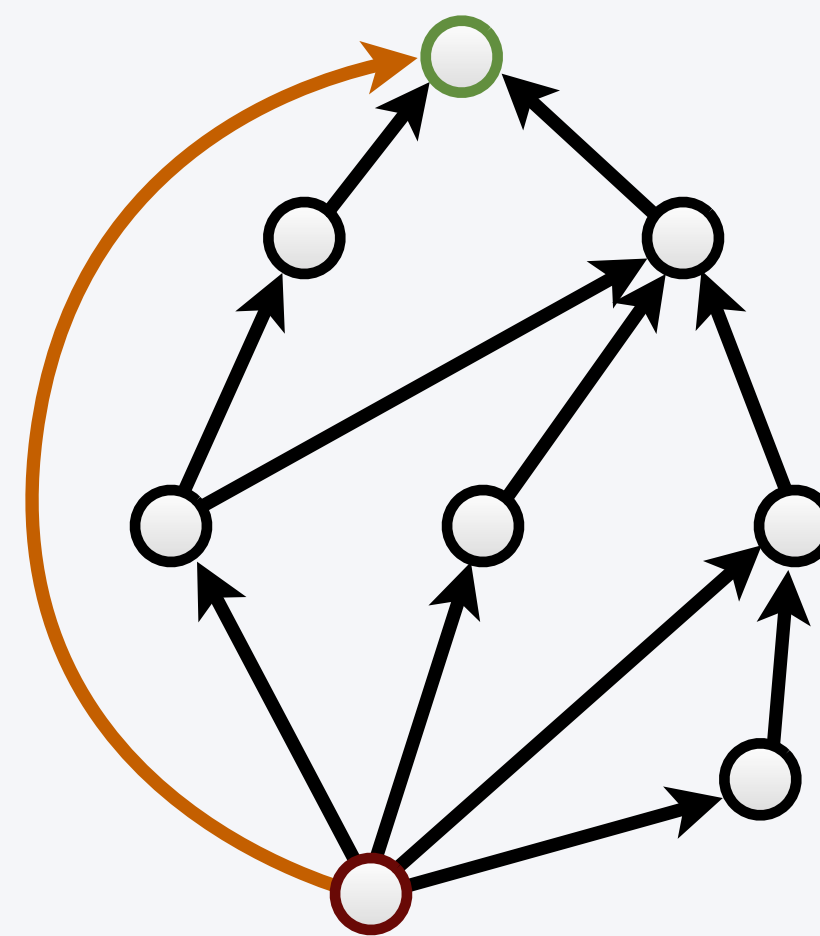
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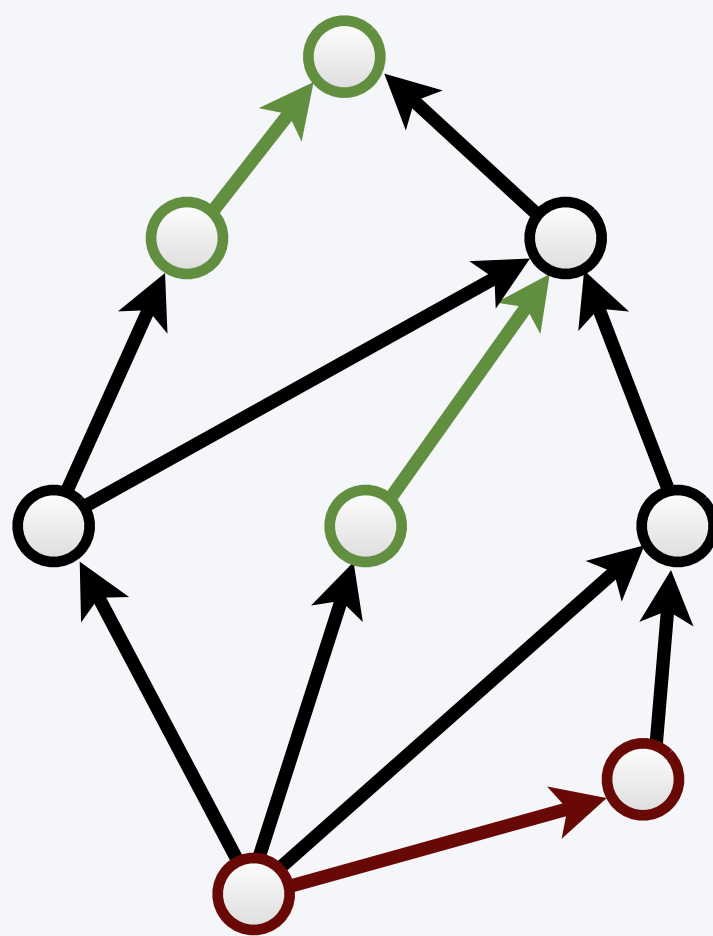
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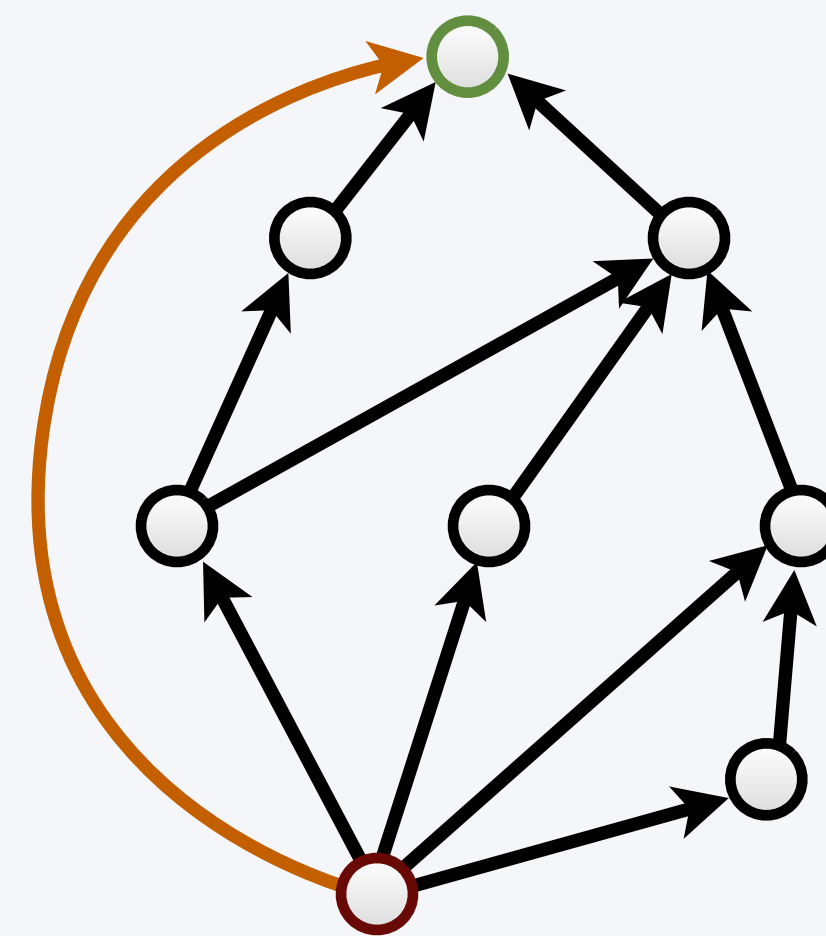
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$\Rightarrow$  *NP*-complete  
[GT95b]

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# Checking Upward Planarity

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Daniel Graf

Special cases in  $P$ :

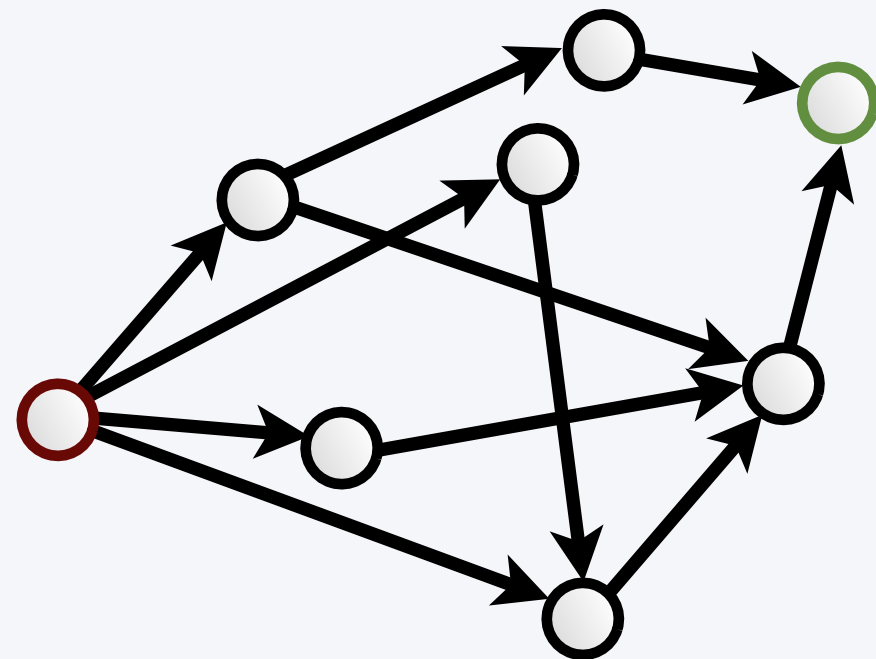


# Checking Upward Planarity

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Daniel Graf

Special cases in  $P$ :

- single source, single sink [DBT88]

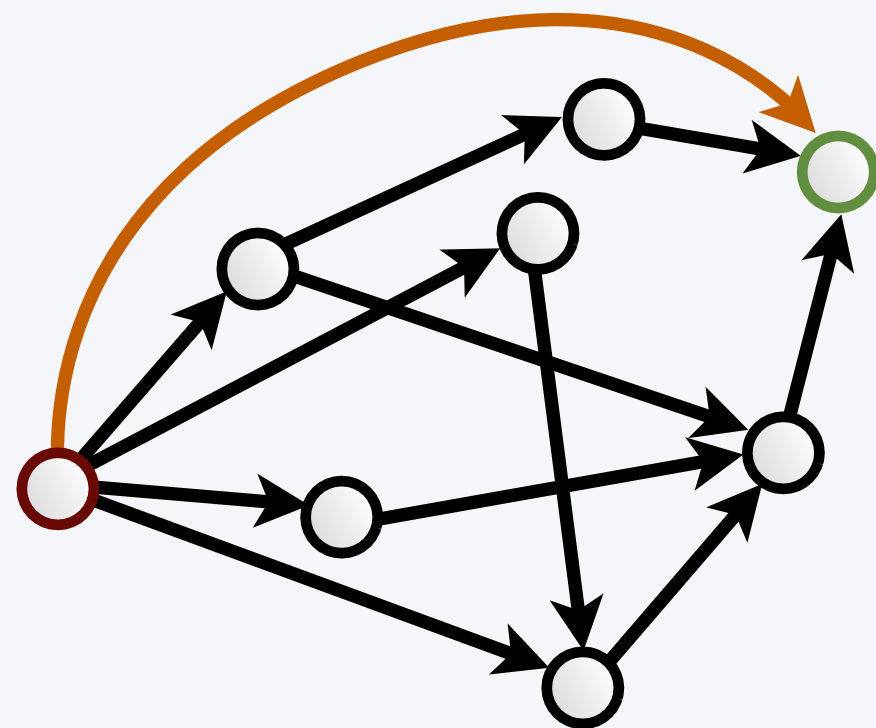


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Daniel Graf

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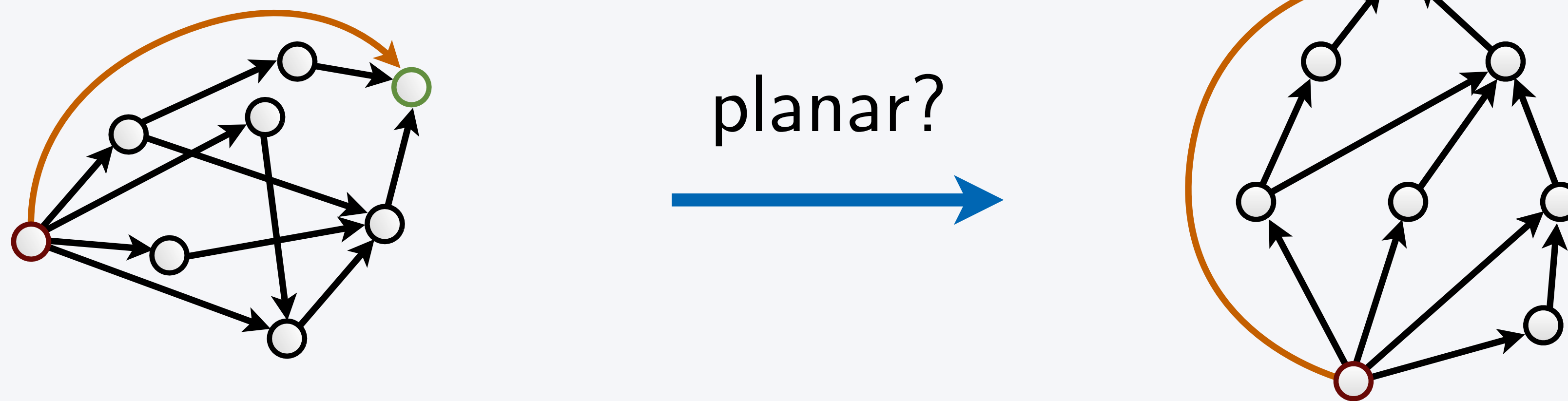


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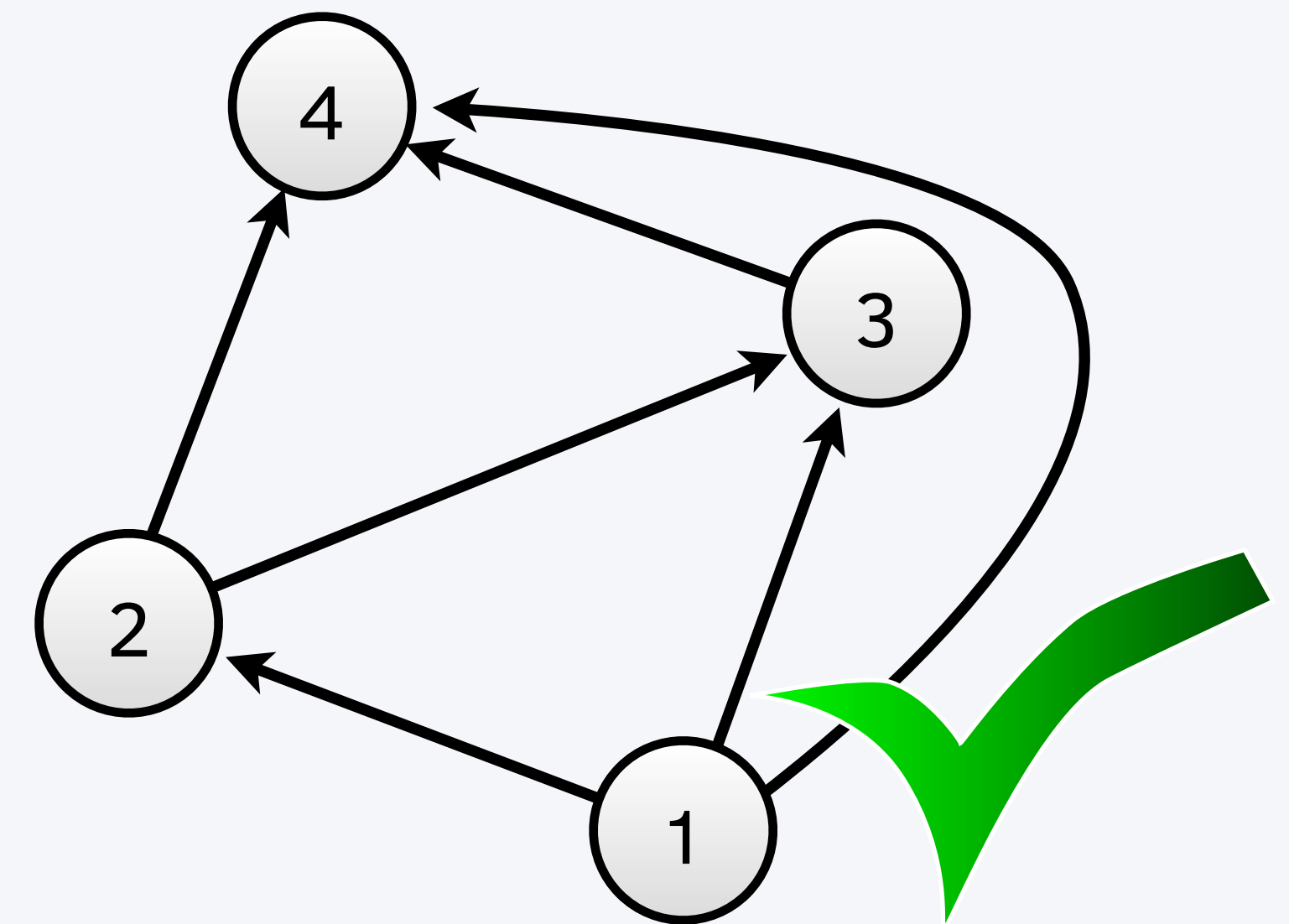
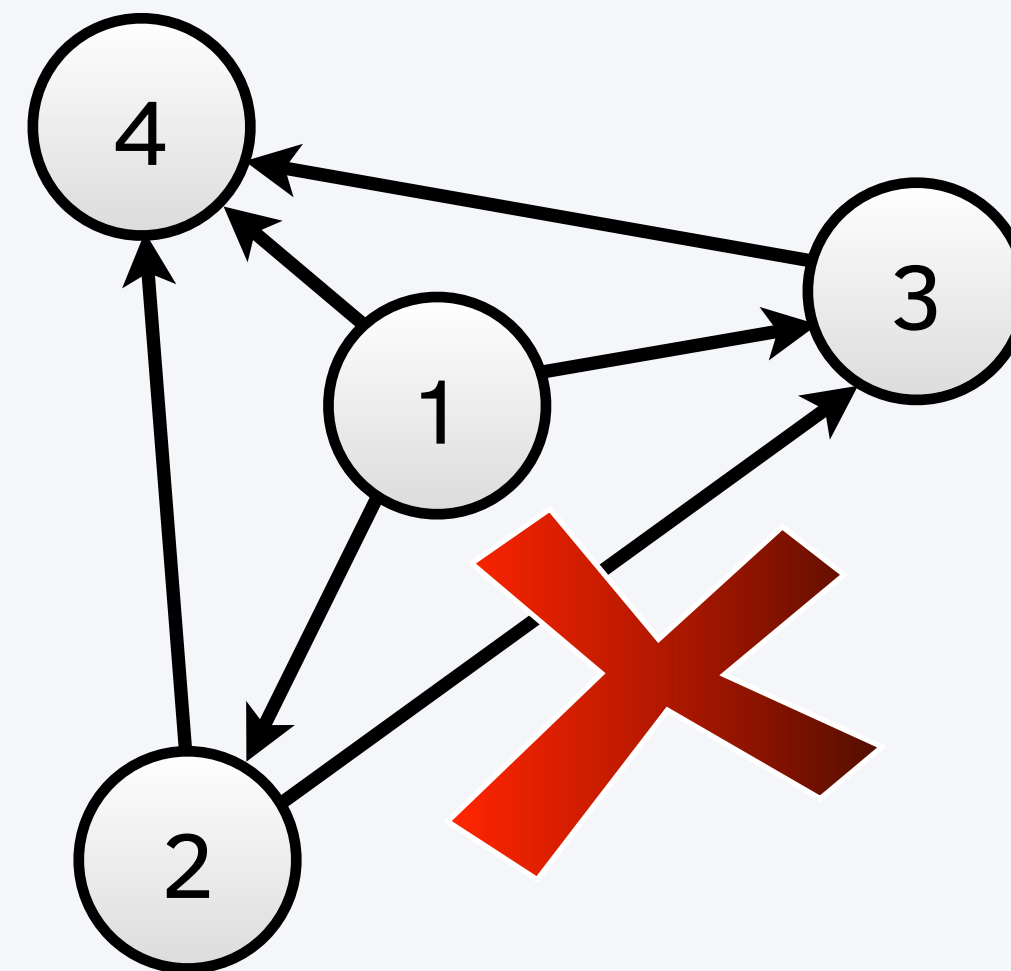


# Checking Upward Planarity

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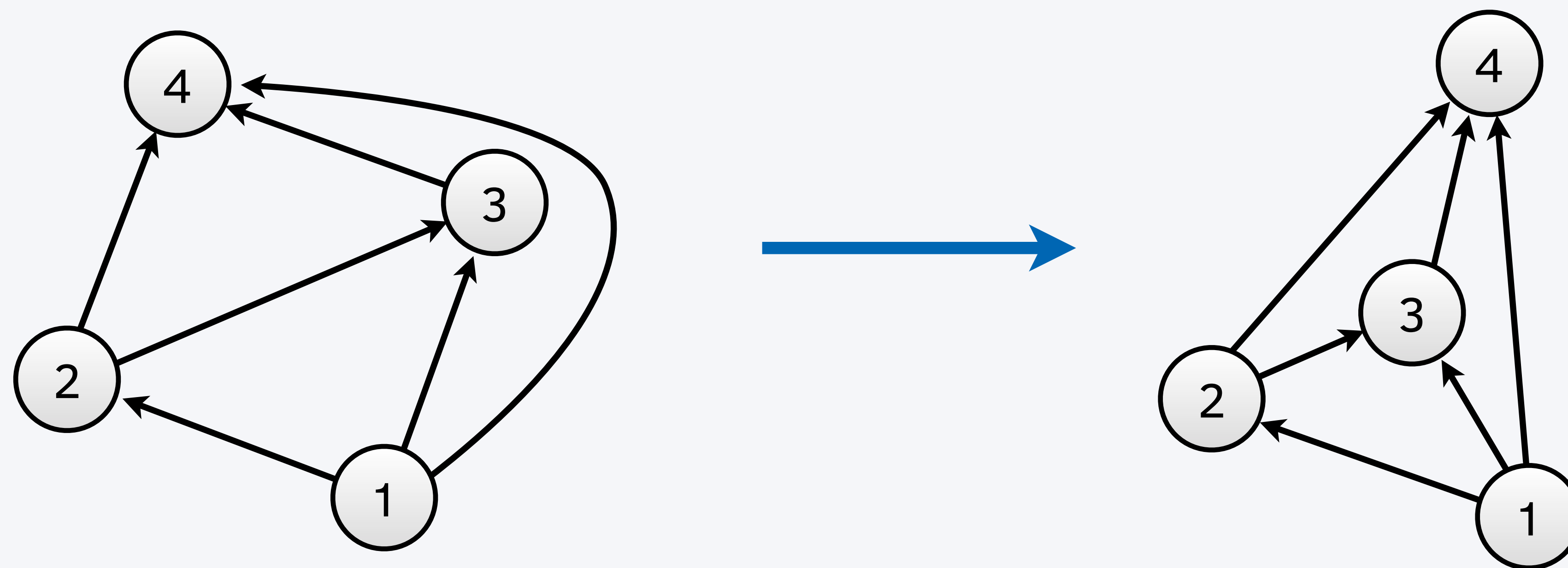
Special cases in  $P$ :

- fixed embedding [BDB91]
- ⇒ maximal planar graphs



# Straight Line Upward Drawing

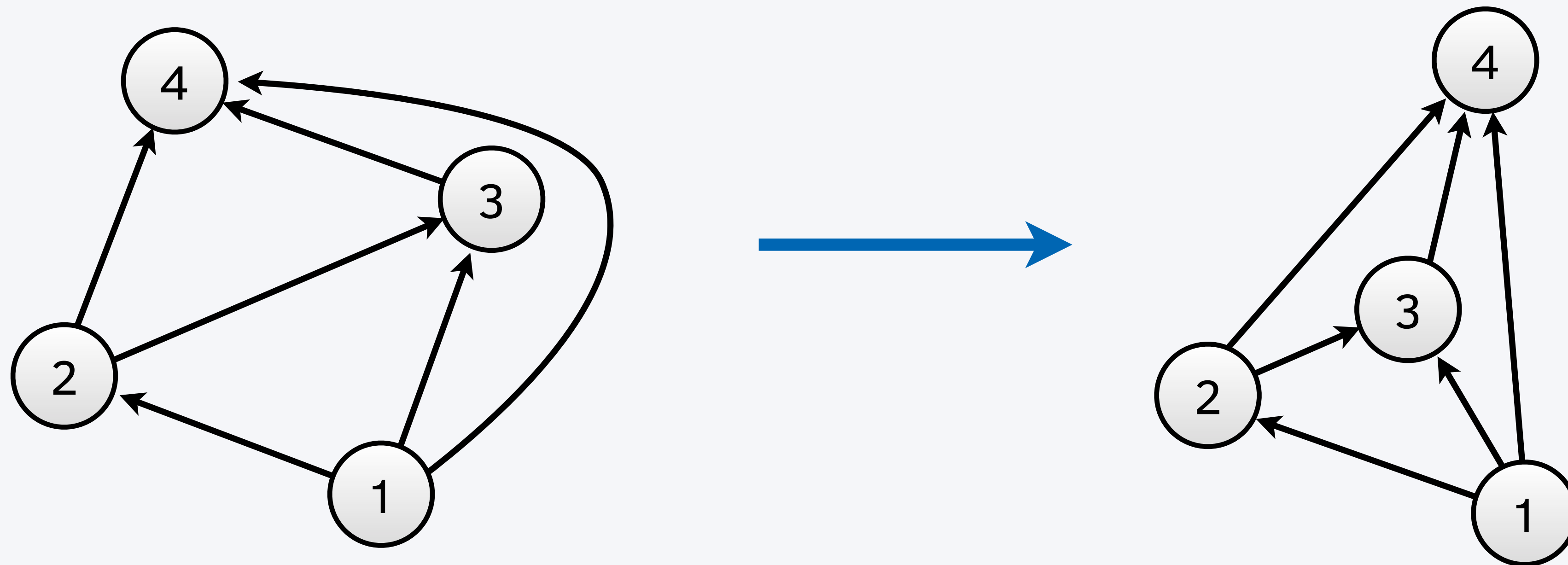
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# Straight Line Upward Drawing

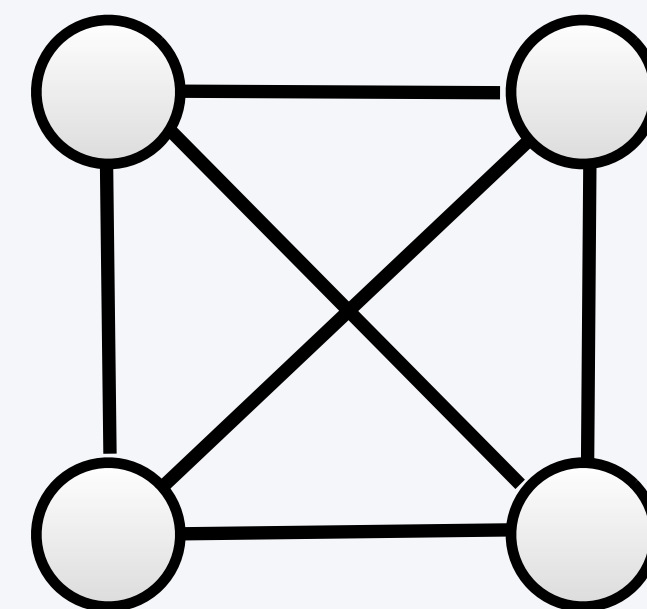
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Always possible, but might need large grid [DBTT92]



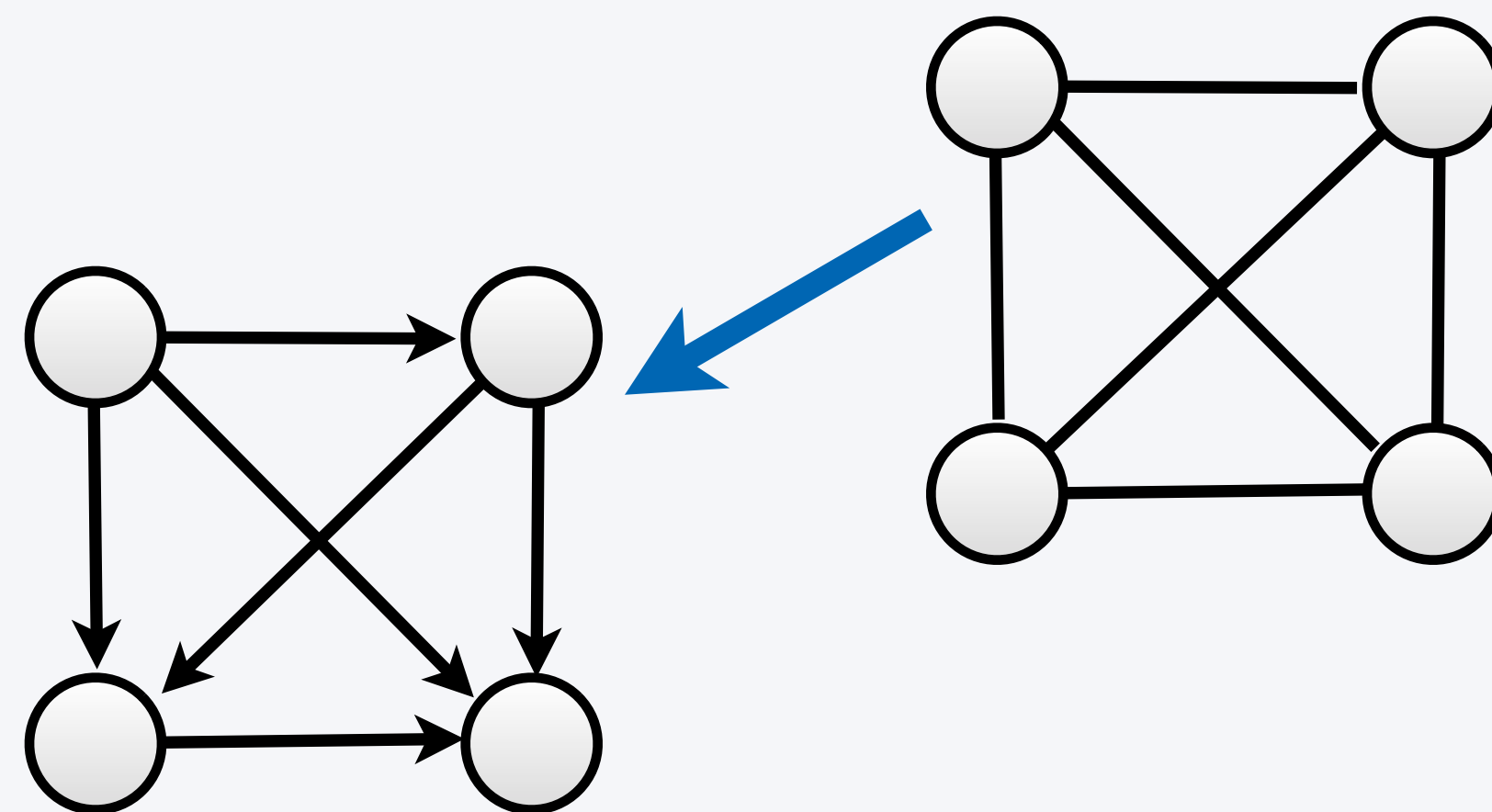
# Upward Planar Orientations

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# Upward Planar Orientations

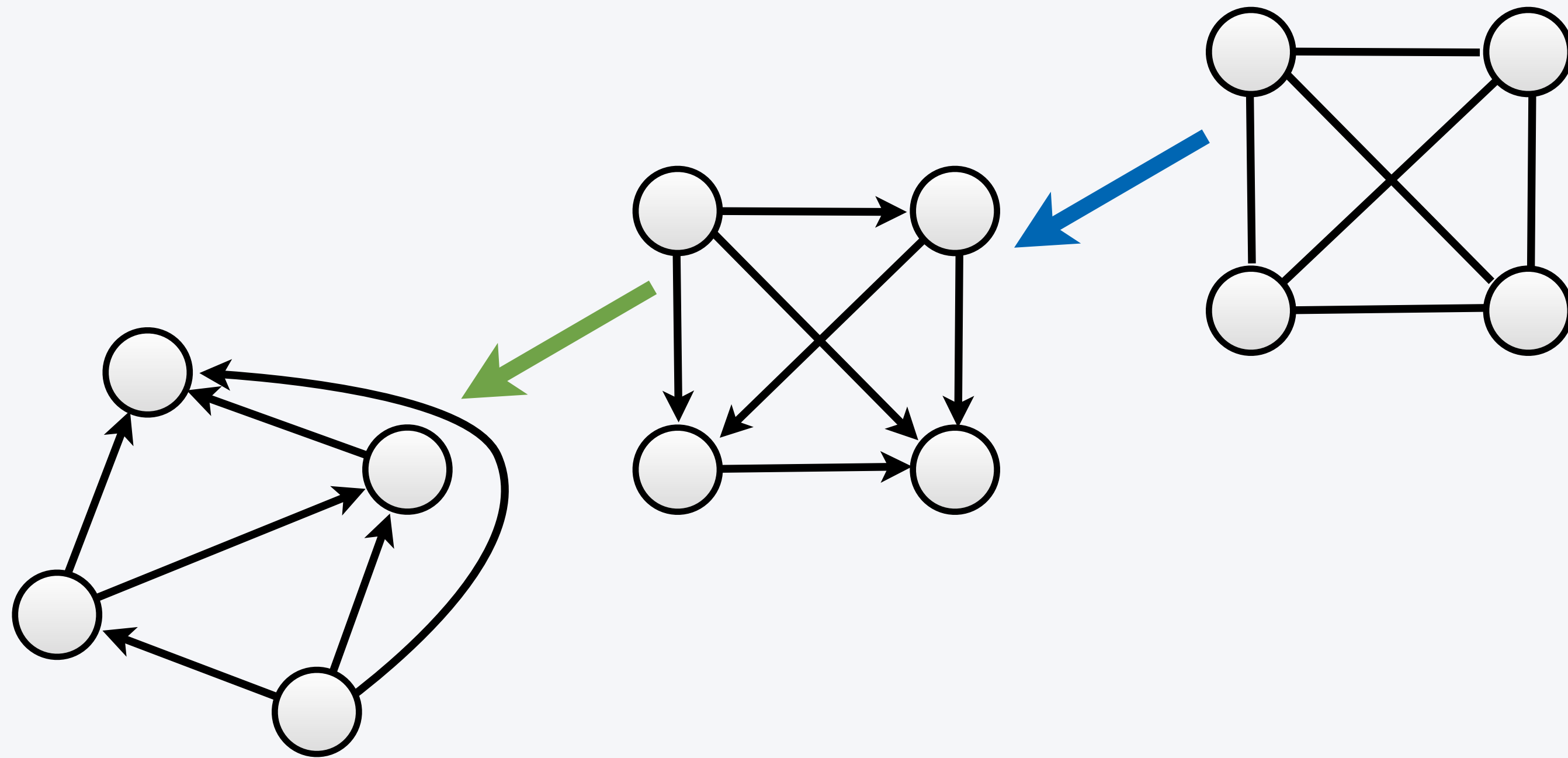
ETH Zürich  
Daniel Graf





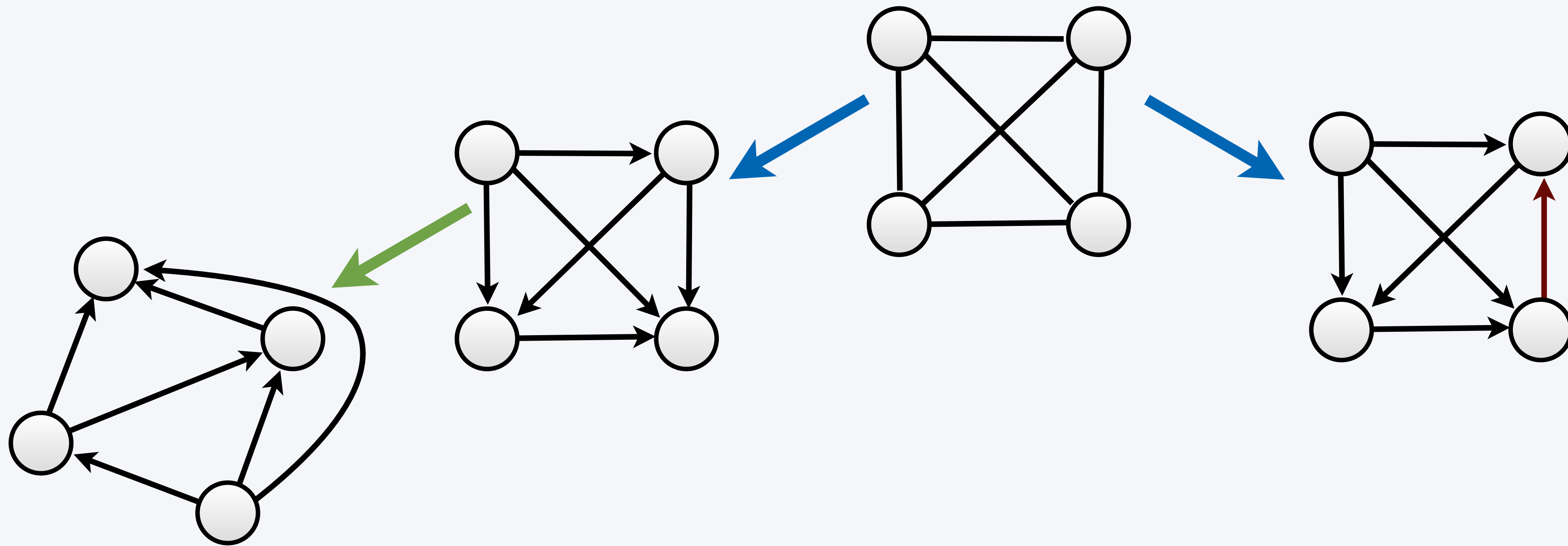
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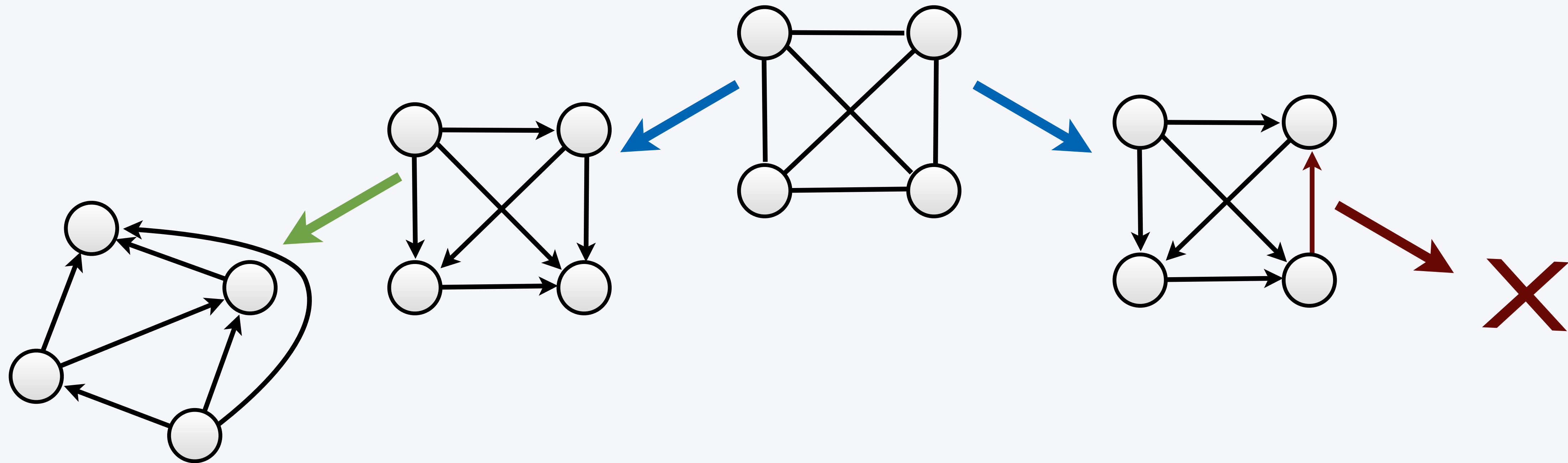
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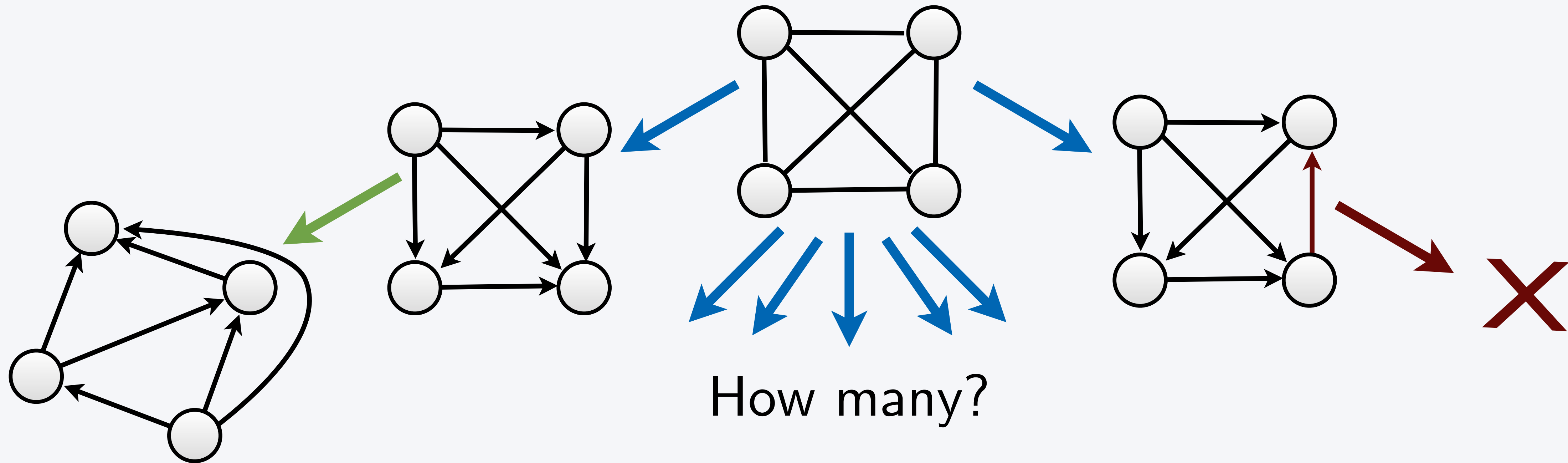
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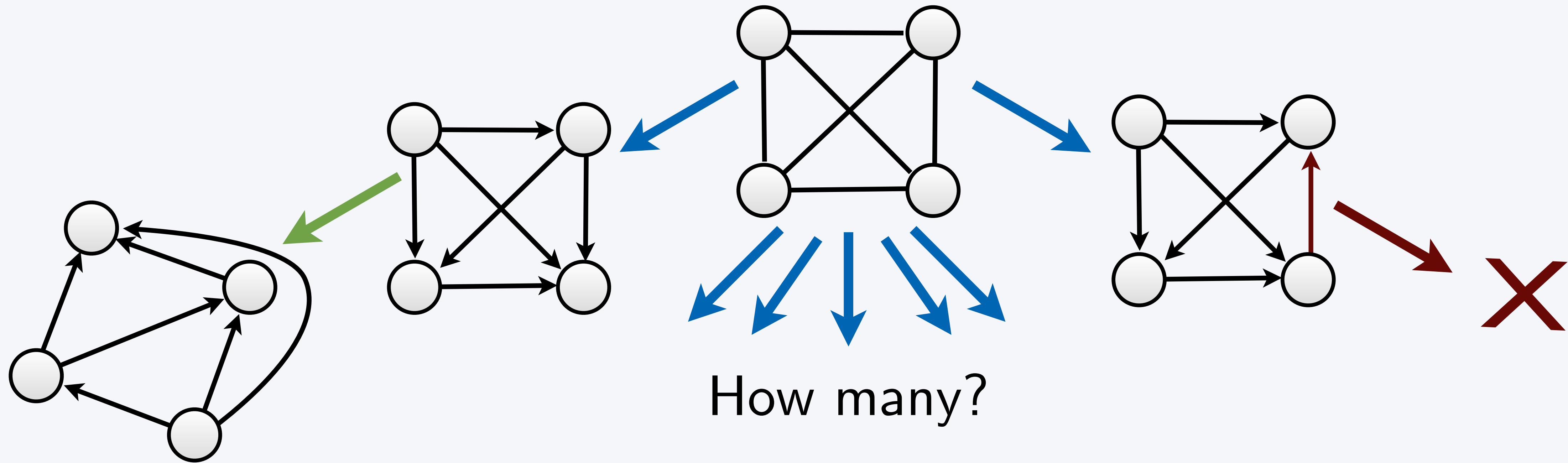
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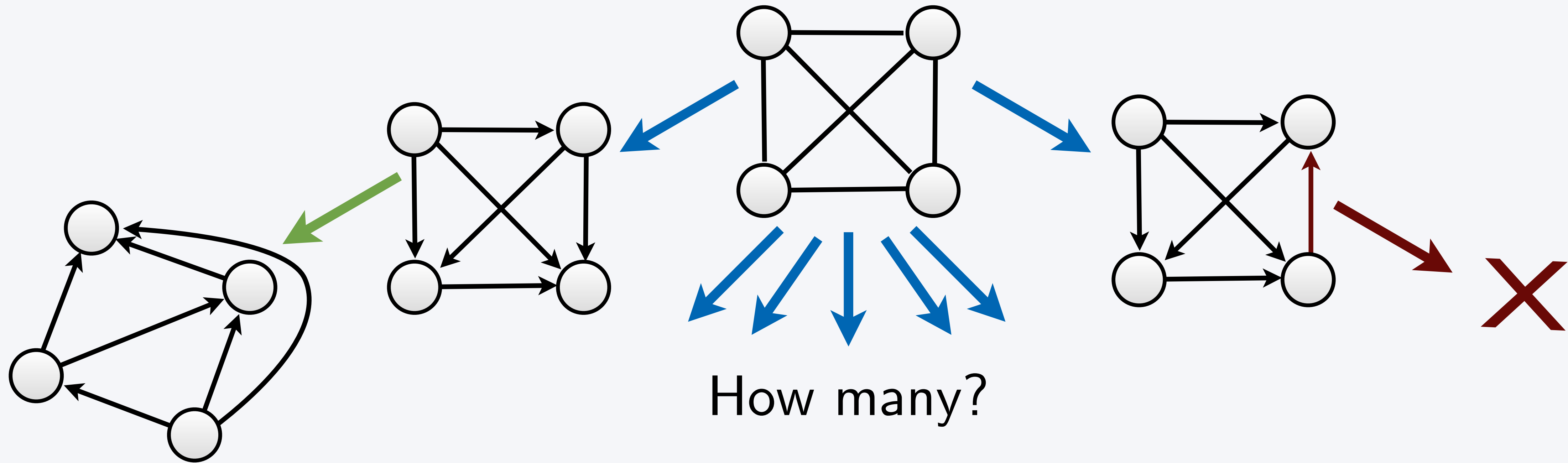


Maximal planar: some graphs with  $\mathcal{O}(2^n)$  orientations

[FGW13] and some graphs with  $\Omega(2.5^n)$  orientations

# Upward Planar Orientations

ETH Zürich  
Daniel Graf



Maximal planar: some graphs with  $\mathcal{O}(2^n)$  orientations  
[FGW13] and some graphs with  $\Omega(2.5^n)$  orientations

Rest: open

# Checking Variations

ETH Zürich  
Daniel Graf

[BDBD98] Paola Bertolazzi, Giuseppe Di Battista, and Walter Didimo. Quasi-upward planarity. In *Graph Drawing*, pages 15–29. Springer, 1998.

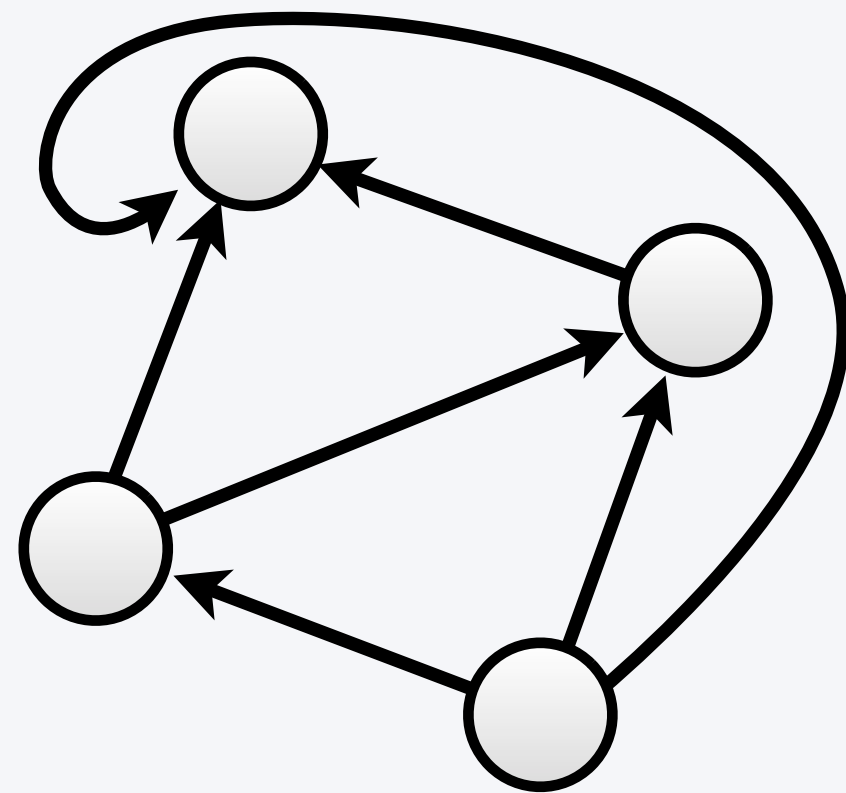
[FKPTW13] Fabrizio Frati, Michael Kaufmann, Janos Pach, Csaba D Toth, and David R Wood. On the upward planarity of mixed plane graphs. In *Graph Drawing*, pages 1–12. Springer, 2013.

[BDP14] Carla Binucci, Walter Didimo, and Maurizio Patrignani. Upward and quasi-upward planarity testing of embedded mixed graphs. *Theoretical Computer Science*, 526:75–89, 2014.

# Checking Variations

ETH Zürich  
Daniel Graf

quasi upward



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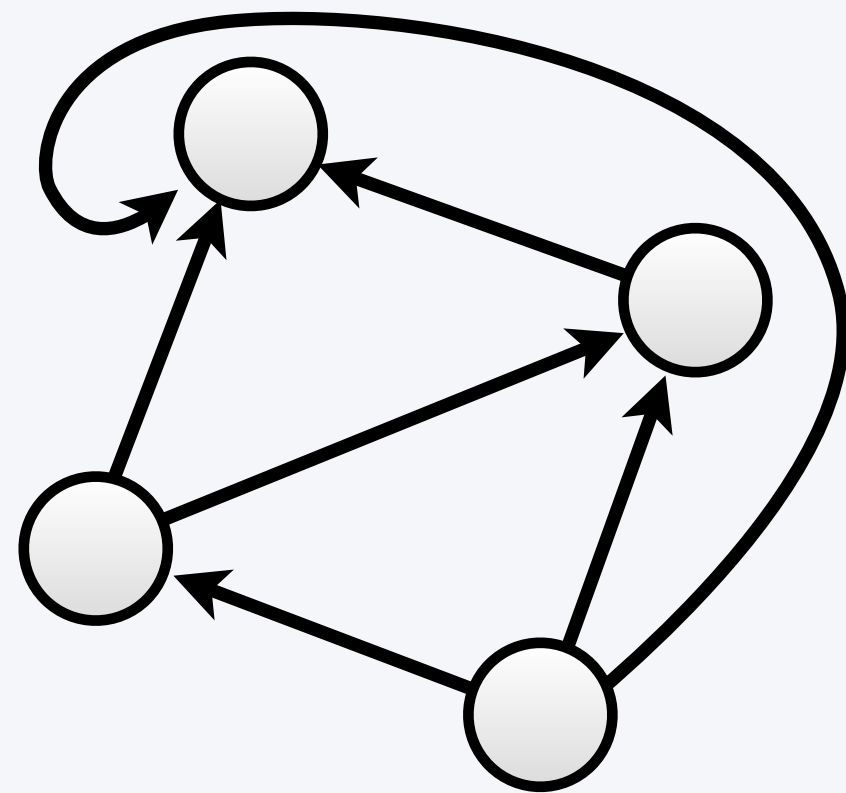
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ETH Zürich  
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quasi upward



[BDBD98] given  $\phi$ , in  $\mathcal{P}$

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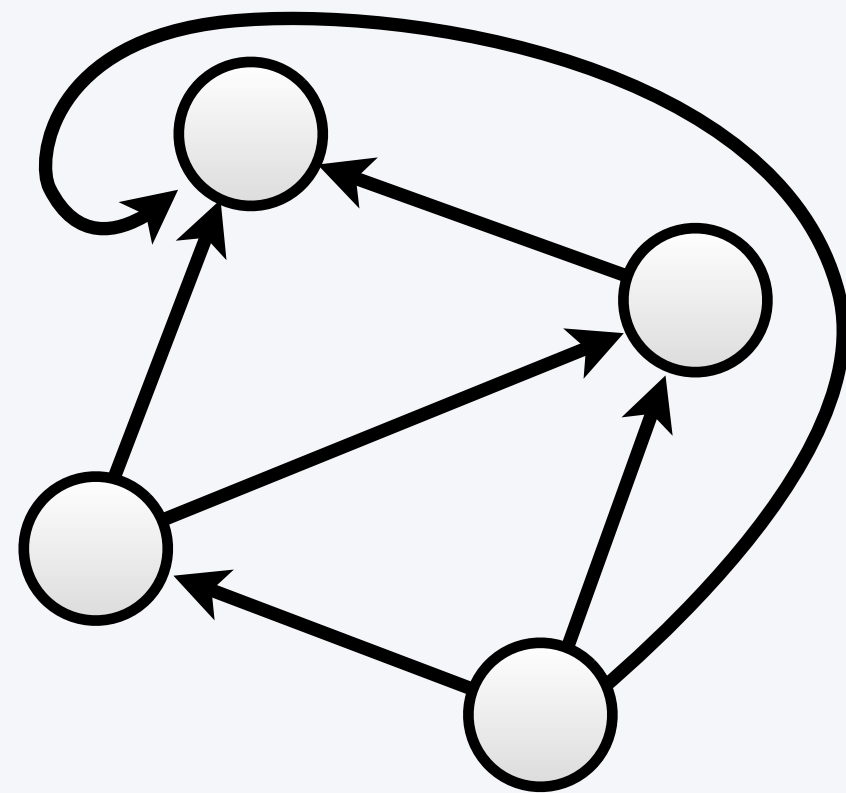
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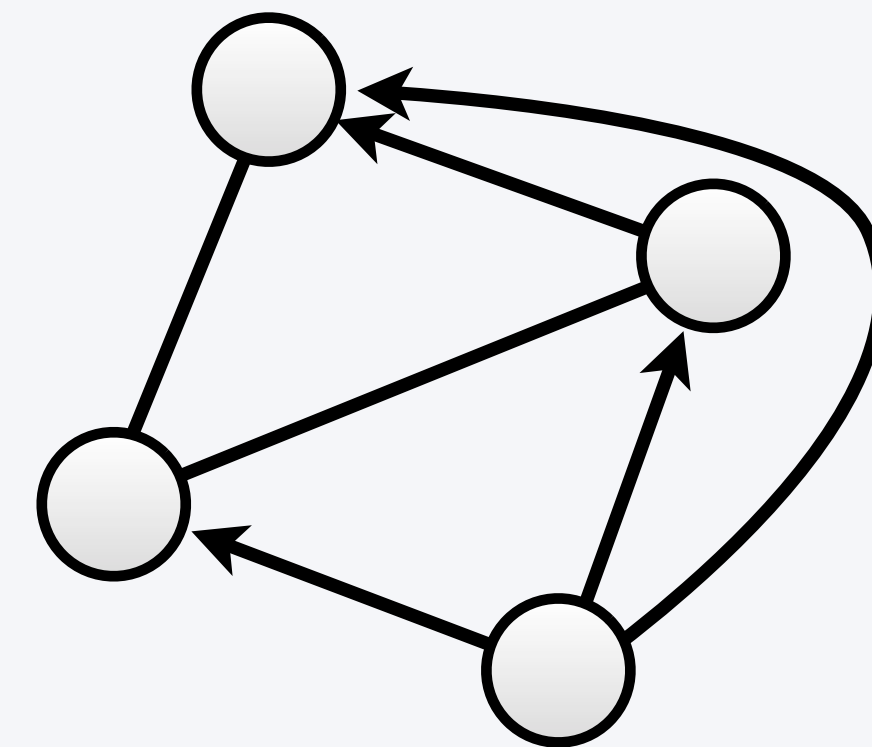
ETH Zürich  
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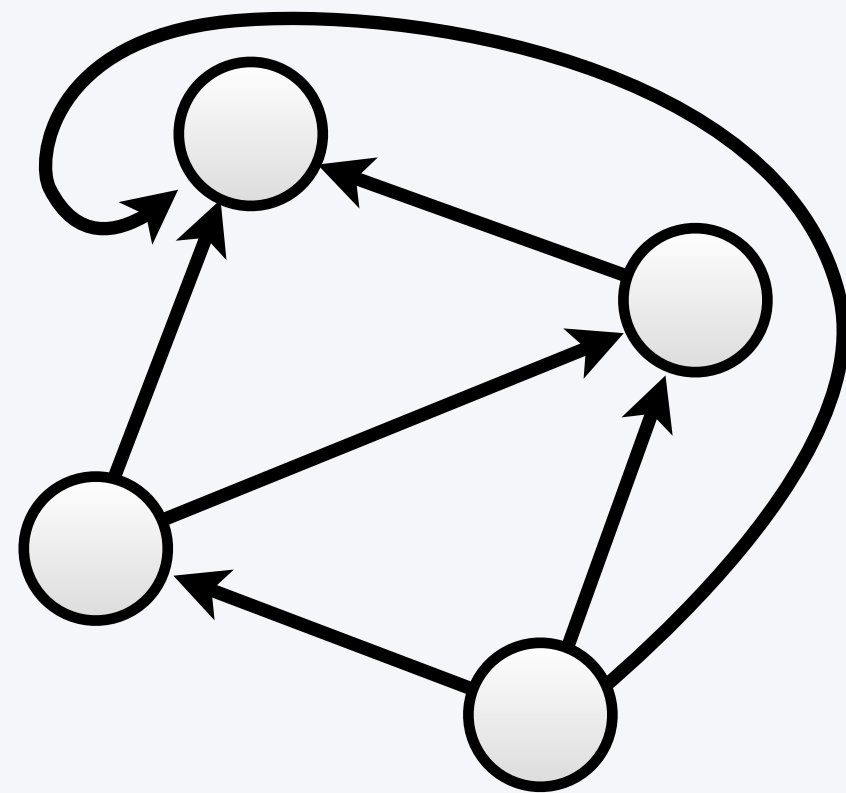
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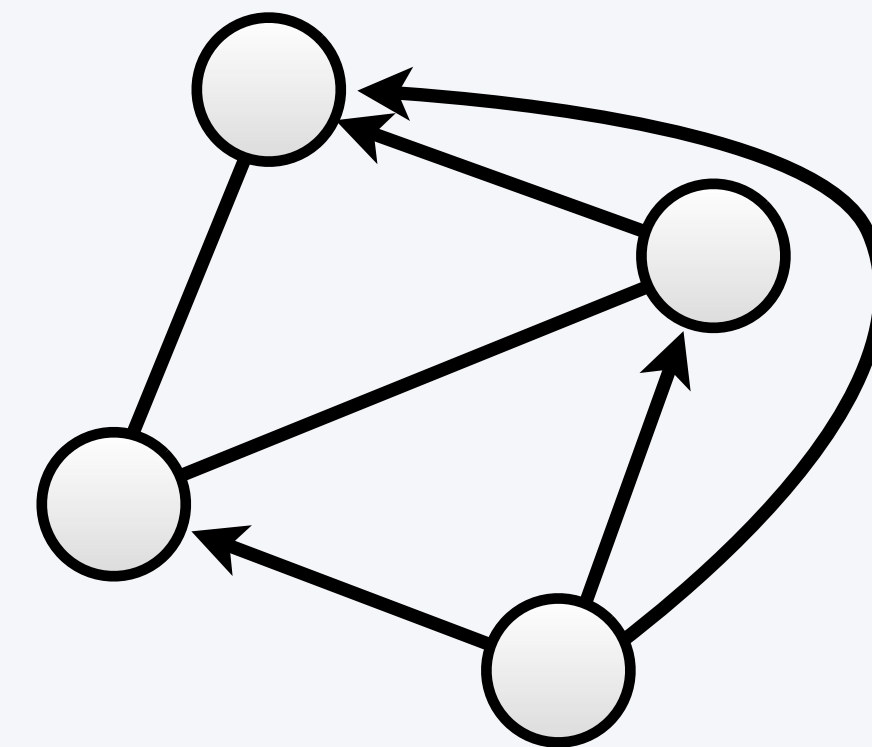
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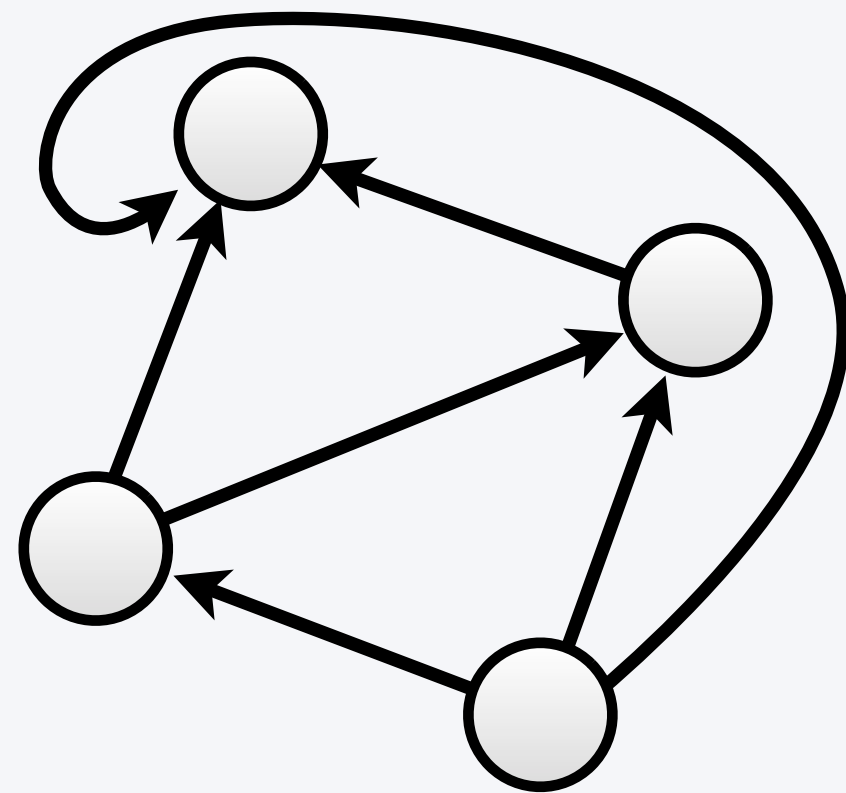
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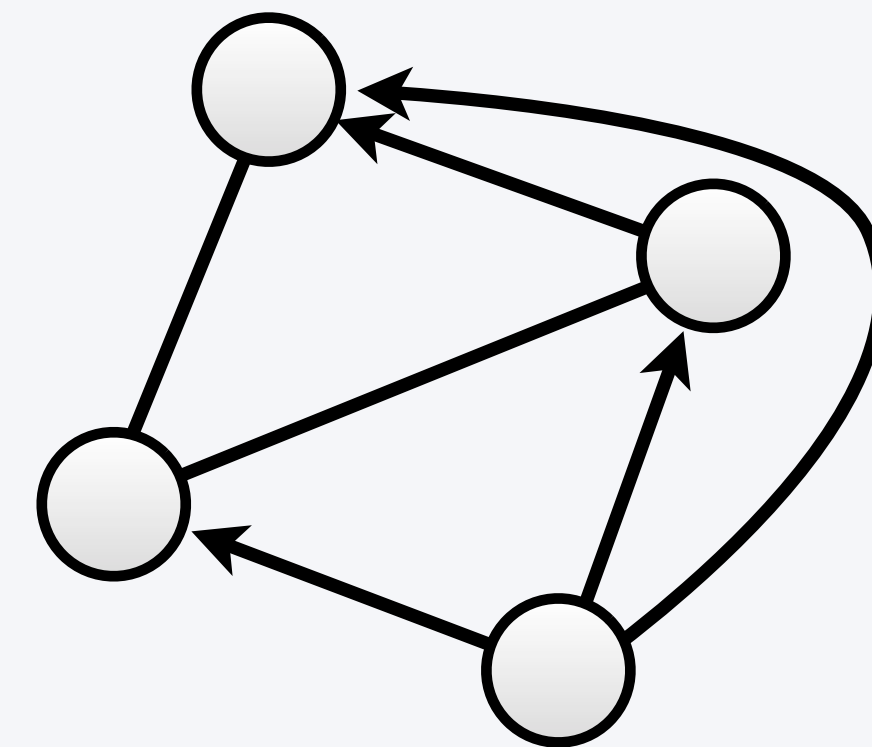
ETH Zürich  
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quasi upward



[BDBD98] given  $\phi$ , in  $\mathcal{P}$

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Open: mixed but fixed  $\phi$ ?

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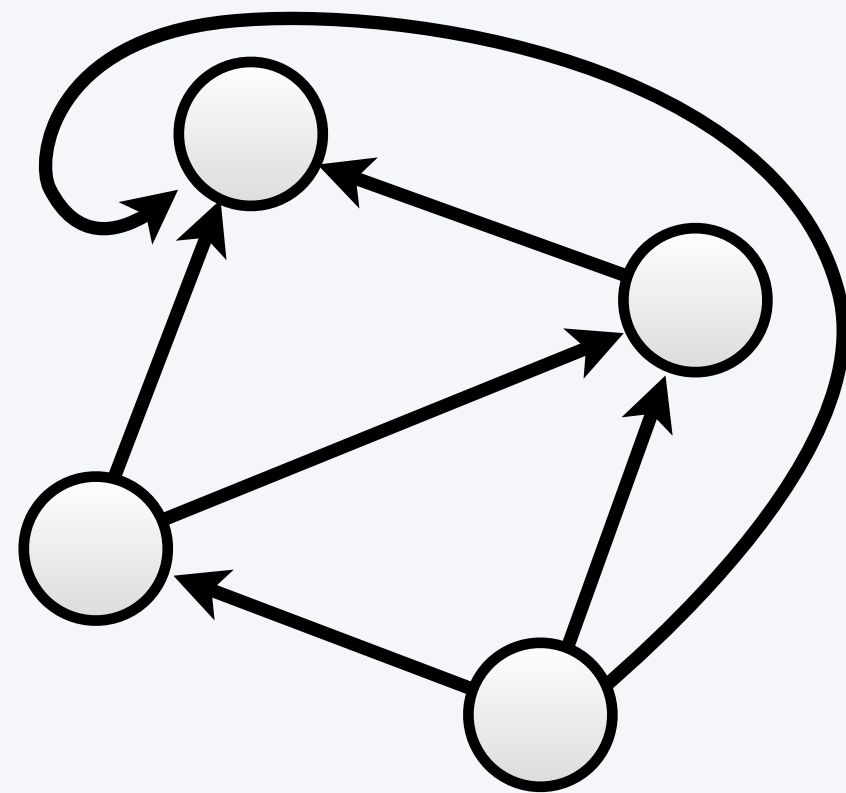
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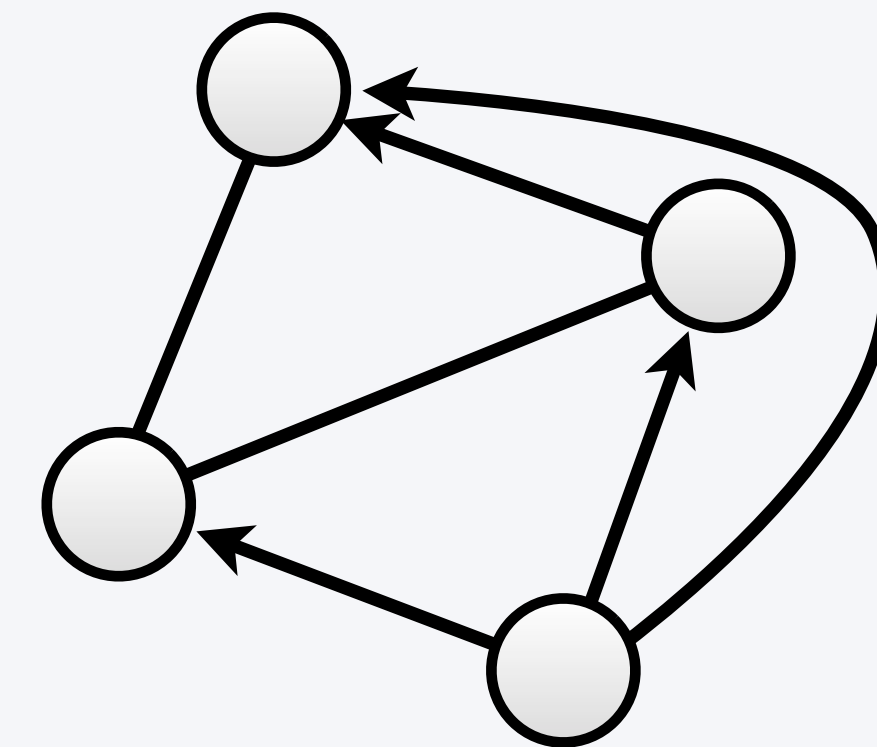
ETH Zürich  
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quasi upward



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- mixed  $\rightarrow$  in  $\mathcal{P}$

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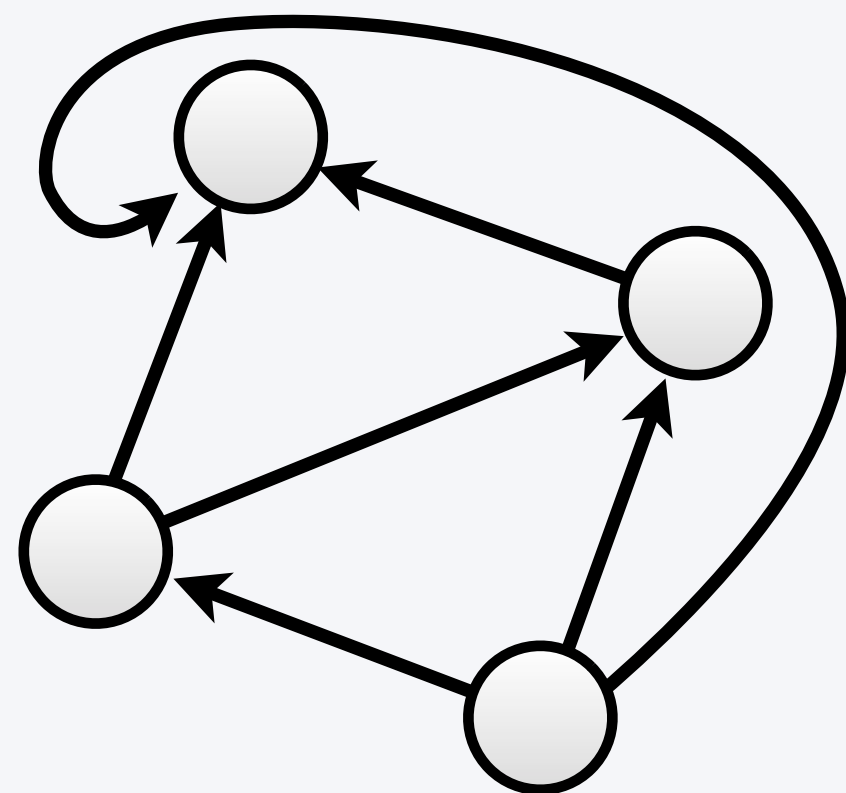
[FKPTW13] Fabrizio Frati, Michael Kaufmann, Janos Pach, Csaba D Toth, and David R Wood. On the upward planarity of mixed plane graphs. In Graph Drawing, pages 1–12. Springer, 2013.

[BDP14] Carla Binucci, Walter Didimo, and Maurizio Patrignani. Upward and quasi-upward planarity testing of embedded mixed graphs. Theoretical Computer Science, 526:75–89, 2014.

# Checking Variations

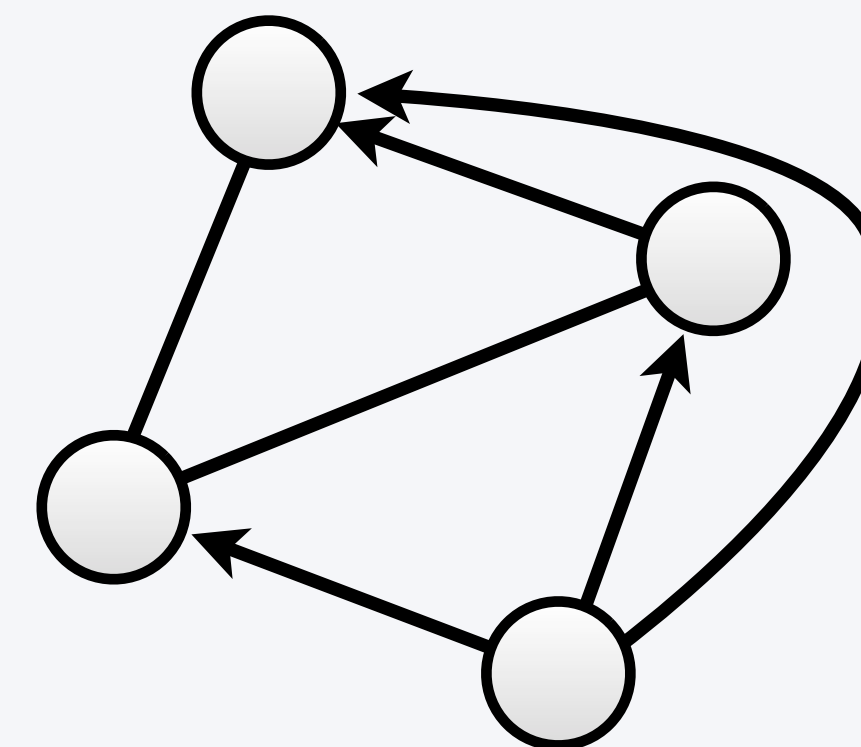
ETH Zürich  
Daniel Graf

quasi upward



[BDBD98] given  $\phi$ , in  $P$

mixed graphs



[FKPTW13] some classes in  $P$

Open: mixed but fixed  $\phi$ ?

- mixed  $\rightarrow$  in  $P$       + quasi  $\rightarrow$  NP-hard [BDP14]

[BDBD98] Paola Bertolazzi, Giuseppe Di Battista, and Walter Didimo. Quasi-upward planarity. In Graph Drawing, pages 15–29. Springer, 1998.

[FKPTW13] Fabrizio Frati, Michael Kaufmann, Janos Pach, Csaba D Toth, and David R Wood. On the upward planarity of mixed plane graphs. In Graph Drawing, pages 1–12. Springer, 2013.

[BDP14] Carla Binucci, Walter Didimo, and Maurizio Patrignani. Upward and quasi-upward planarity testing of embedded mixed graphs. Theoretical Computer Science, 526:75–89, 2014.