Upward Planarity
Drawing Directed Graphs
Drawing Directed Graphs

upward

(toposort)
Drawing Directed Graphs

upward

(toposort)

planar

(Boyer Myrvold)
Drawing Directed Graphs

upward planar?

upward (toposort)

planar (Boyer Myrvold)
Drawing Directed Graphs

upward

(toposort)

upward planar?

(Boyer Myrvold)

planar
Acyclicity is not enough
Checking Upward Planarity

Checking Upward Planarity

$NP$-hard in general [GT95a]

NP-hard in general [GT95a]

but: nice characterization [DBT88],[K87]
Checking Upward Planarity

$NP$-hard in general [GT95a]
but: nice characterization [DBT88],[K87]

upward planar

Checking Upward Planarity

\[ \text{NP-hard in general [GT95a]} \]

but: nice characterization [DBT88],[K87]

upward planar

\[ \text{s-t-planar} \]

Checking Upward Planarity

*NP*-hard in general [GT95a]

but: nice characterization [DBT88],[K87]

| up
|ward planar | s-t-planar |
|---|---|---|
| is spanning | is spanning | is spanning |
| subgraph of | subgraph of | subgraph of |

Checking Upward Planarity

$NP$-hard in general [GT95a]

but: nice characterization [DBT88],[K87]

upward planar

is spanning subgraph of

$s$-$t$-planar


Checking Upward Planarity

$NP$-hard in general [GT95a]

but: nice characterization [DBT88],[K87]

$\Rightarrow NP$-complete [GT95b]


Special cases in $P$:

Special cases in $P$:

- single source, single sink [DBT88]

Special cases in $P$:

- single source, single sink [DBT88]

Special cases in $P$:

- single source, single sink [DBT88]

Checking Upward Planarity

Special cases in $P$:

- fixed embedding [BDB91]

$\Rightarrow$ maximal planar graphs

Always possible, but might need large grid [DBTT92]

Upward Planar Orientations

Upward Planar Orientations

Upward Planar Orientations

Upward Planar Orientations

Upward Planar Orientations

Upward Planar Orientations

How many?

Maximal planar: some graphs with $\mathcal{O}(2^n)$ orientations

[FGW13] and some graphs with $\Omega(2.5^n)$ orientations

Upward Planar Orientations

Maximal planar: some graphs with $O(2^n)$ orientations

[FGW13] and some graphs with $\Omega(2.5^n)$ orientations

Rest: open


Checking Variations

quasi upward


Checking Variations

quasi upward

[BDBD98] given \( \phi \), in \( P \)

Checking Variations

quasi upward

mixed graphs

[BDBD98] given $\phi$, in $P$


Checking Variations

quasi upward

mixed graphs

[BDBD98] given $\phi$, in $P$

[FKPTW13] some classes in $P$


Checking Variations

quasi upward

mixed graphs

[BDBD98] given $\phi$, in $P$

[FKPTW13] some classes in $P$

Open: mixed but fixed $\phi$?


Checking Variations

quasi upward

mixed graphs

[BDBD98] given $\phi$, in $P$

[FKPTW13] some classes in $P$

Open: mixed but fixed $\phi$?

- mixed $\rightarrow$ in $P$


Checking Variations

[BDBD98] given $\phi$, in $P$

[FKPTW13] some classes in $P$

Open: mixed but fixed $\phi$?

- mixed $\rightarrow$ in $P$

+ quasi $\rightarrow$ NP-hard [BDP14]

