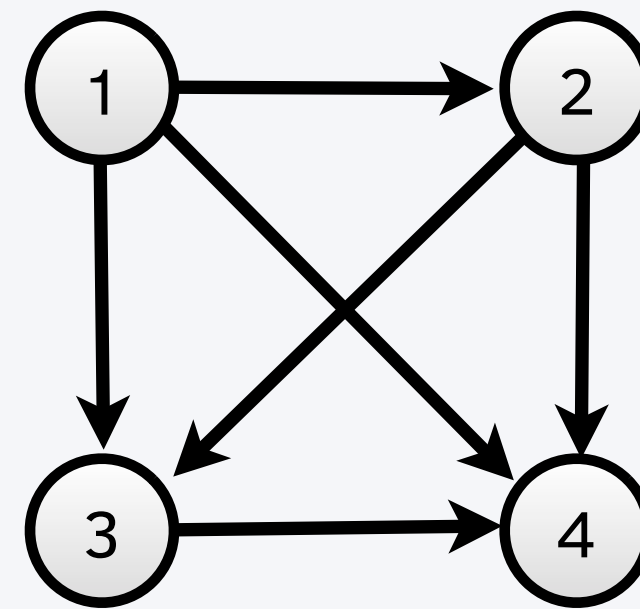


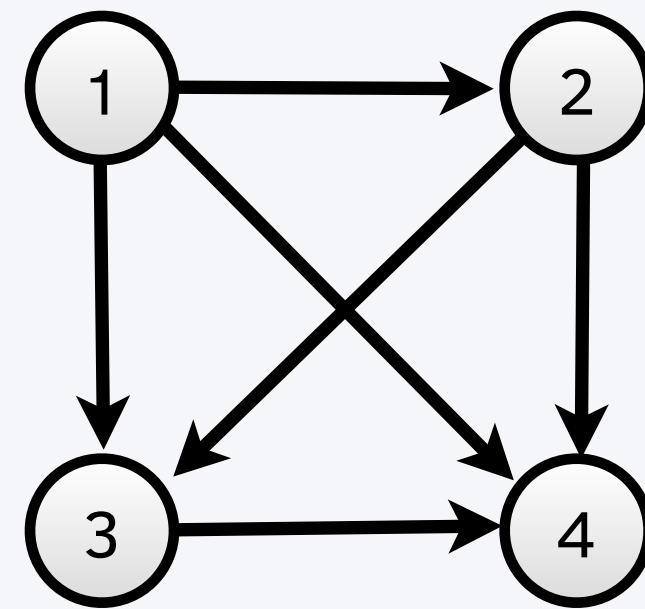
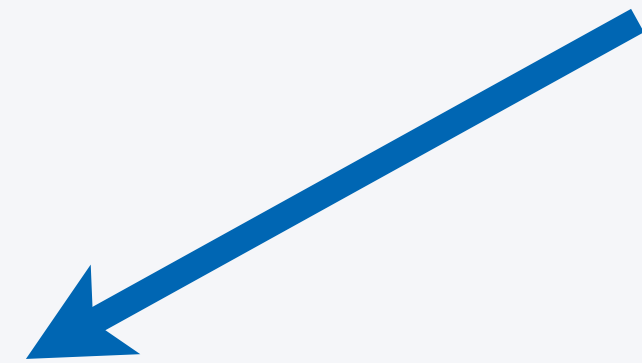
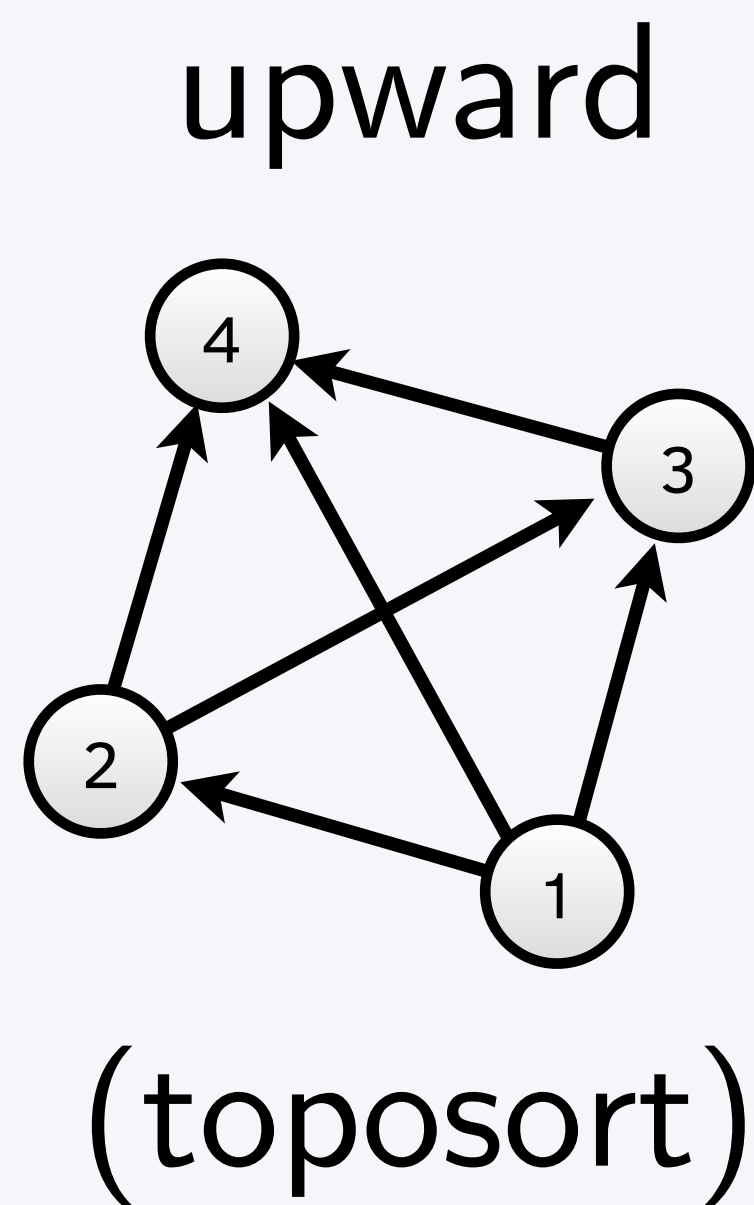
Drawing Directed Graphs

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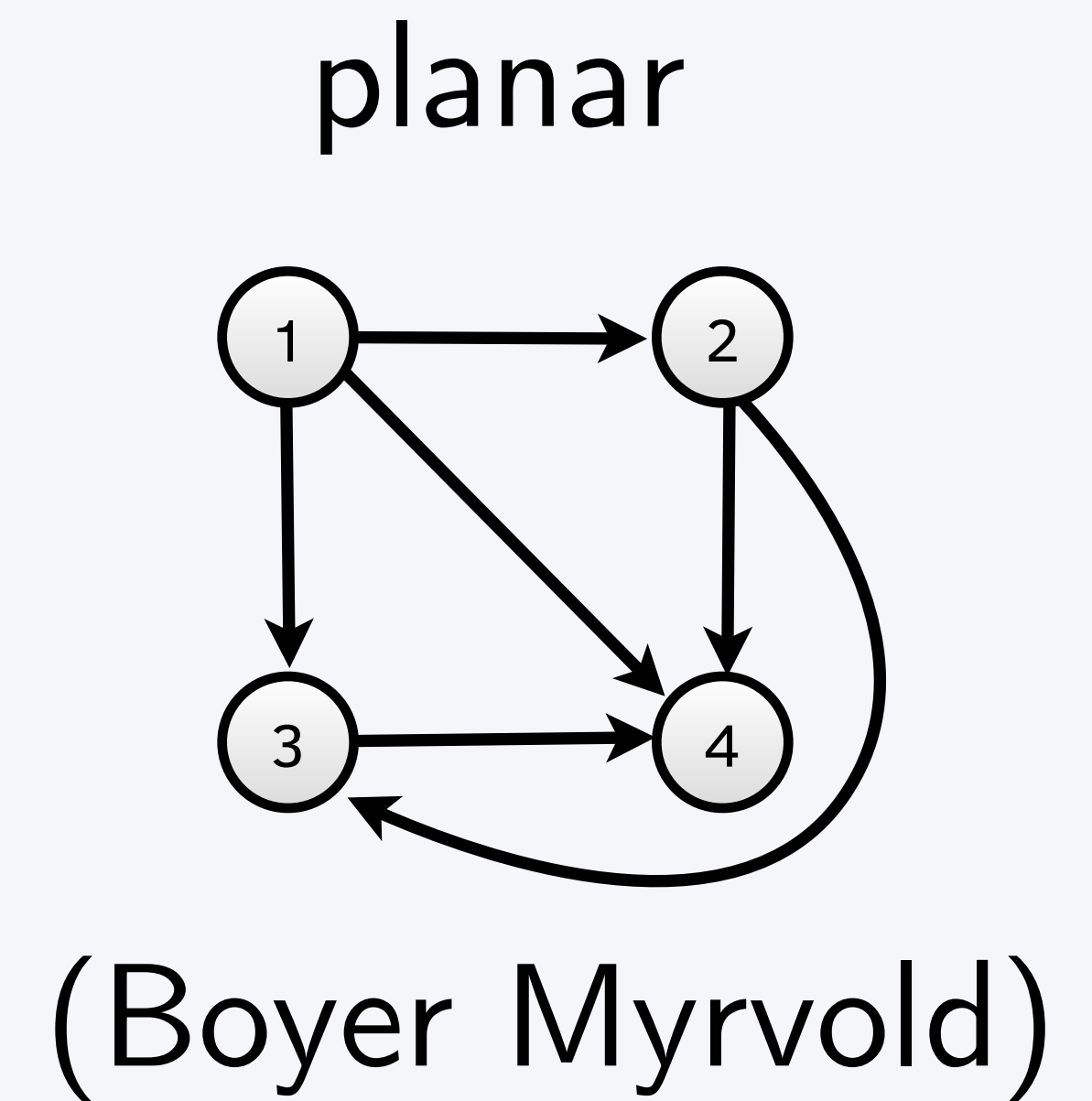
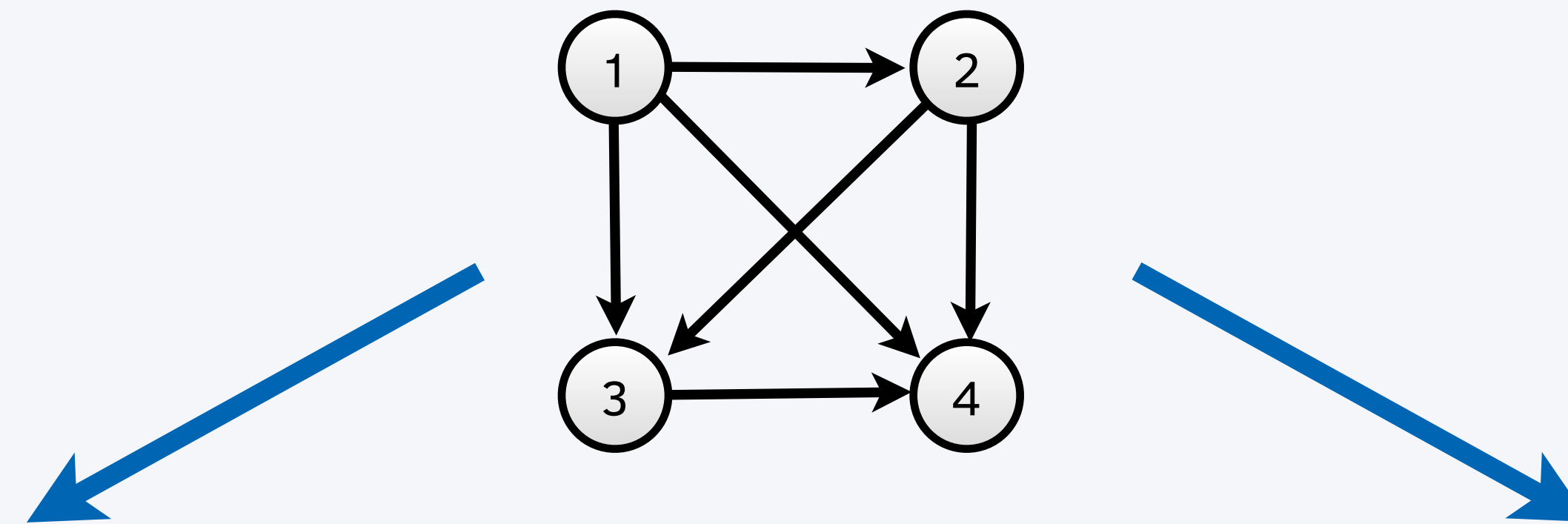
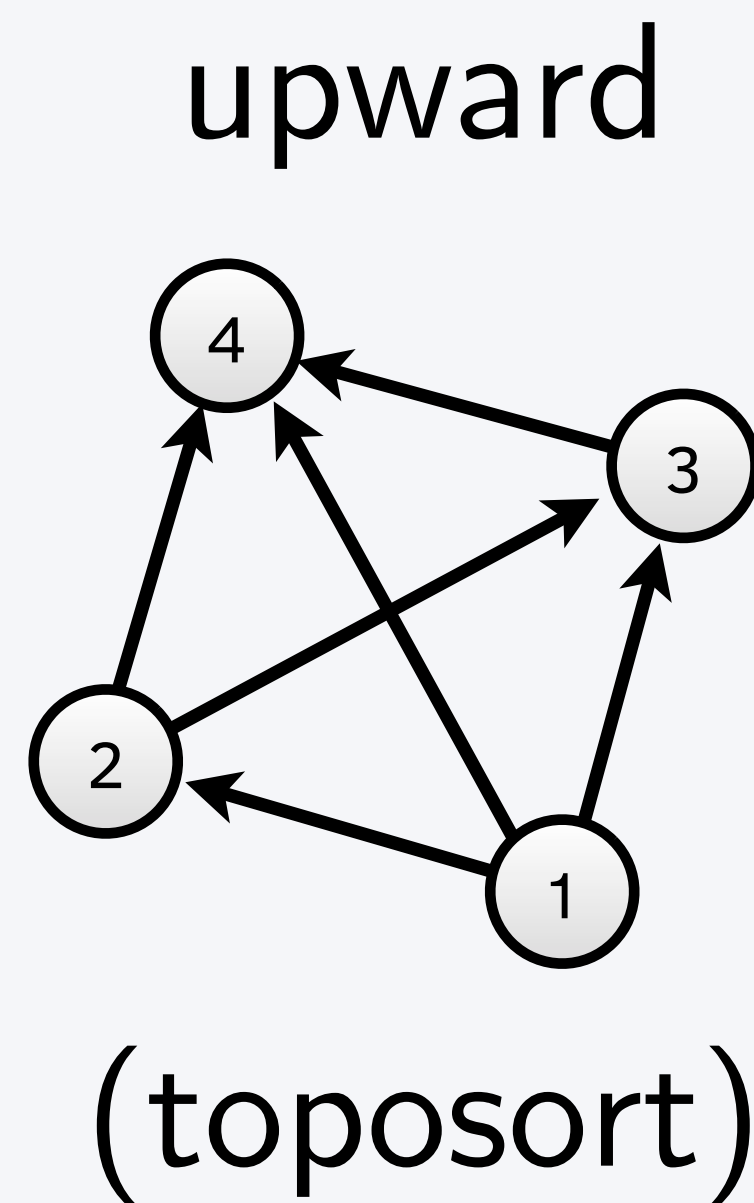
Drawing Directed Graphs

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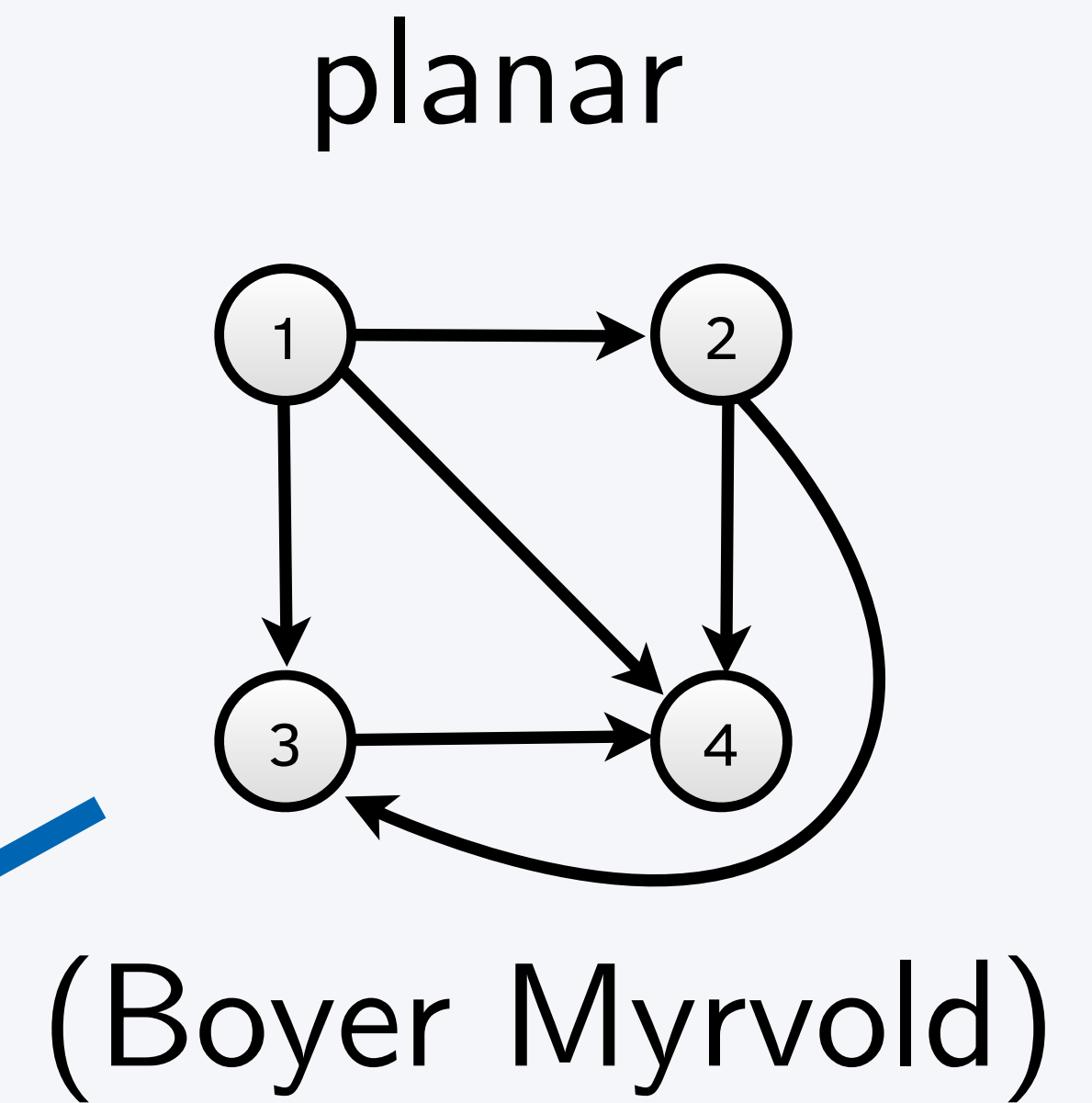
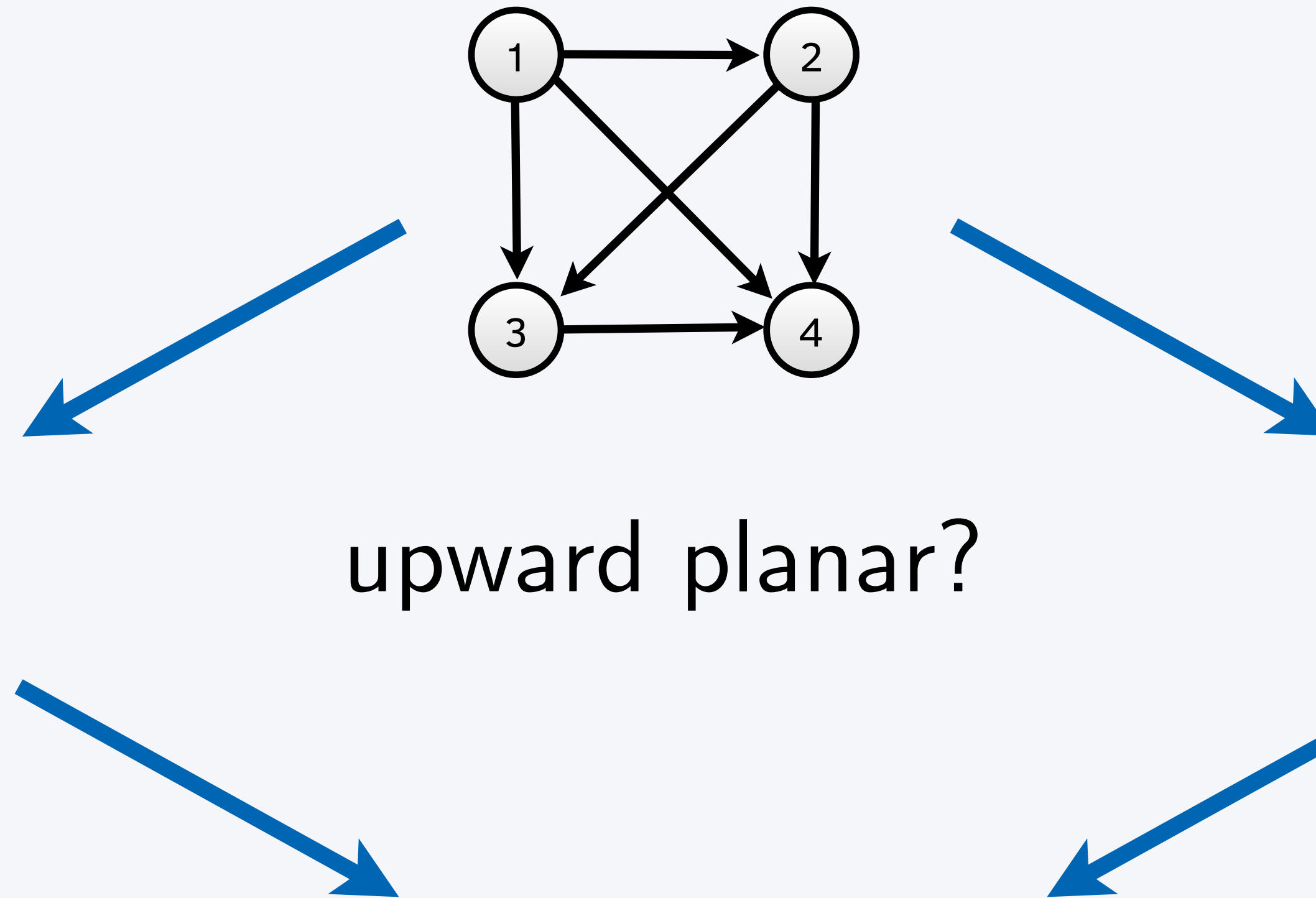
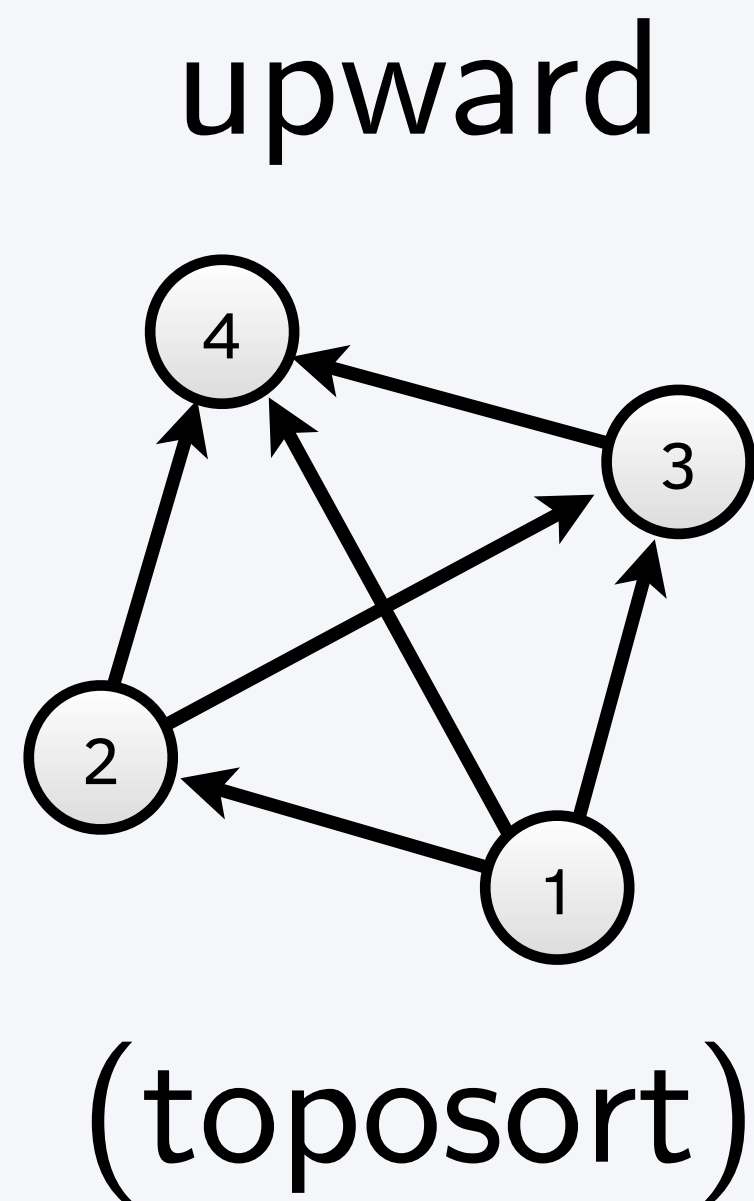
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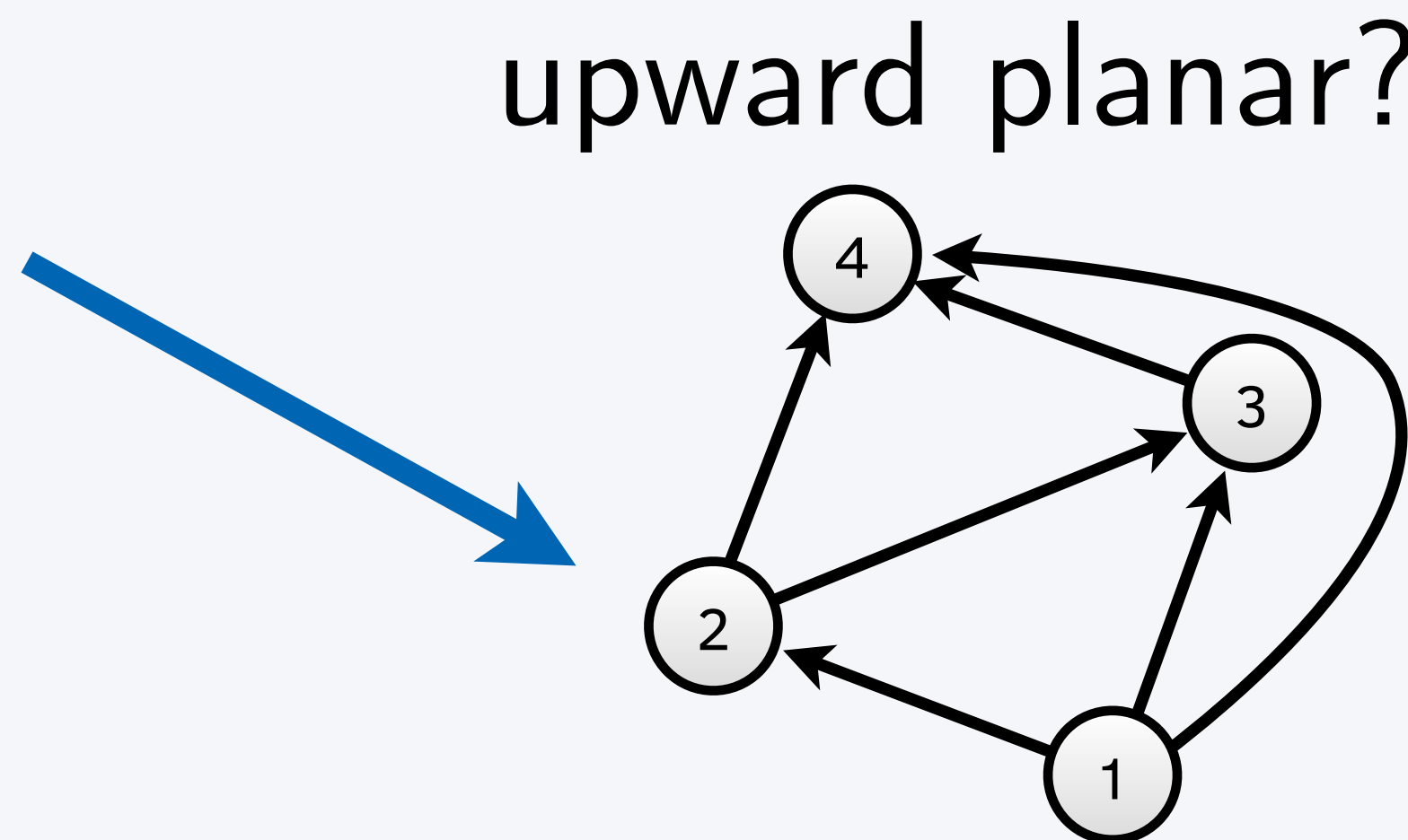
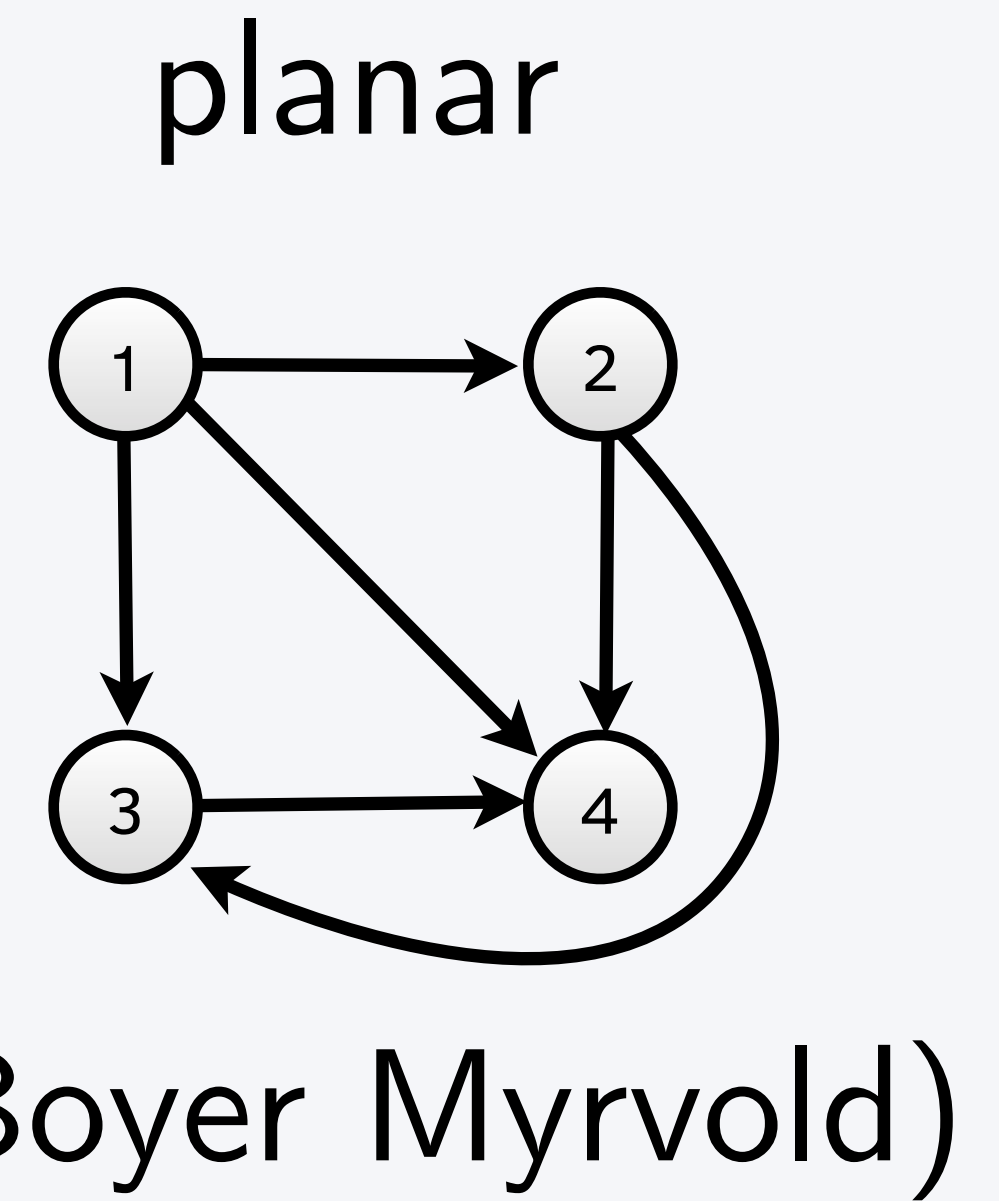
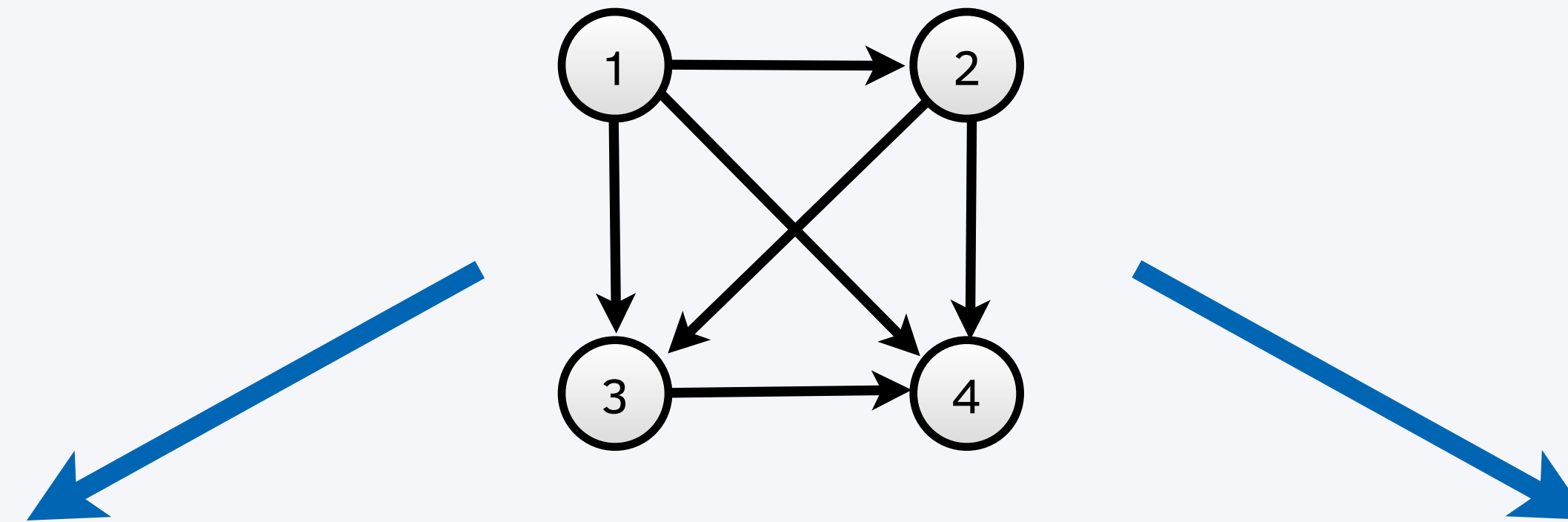
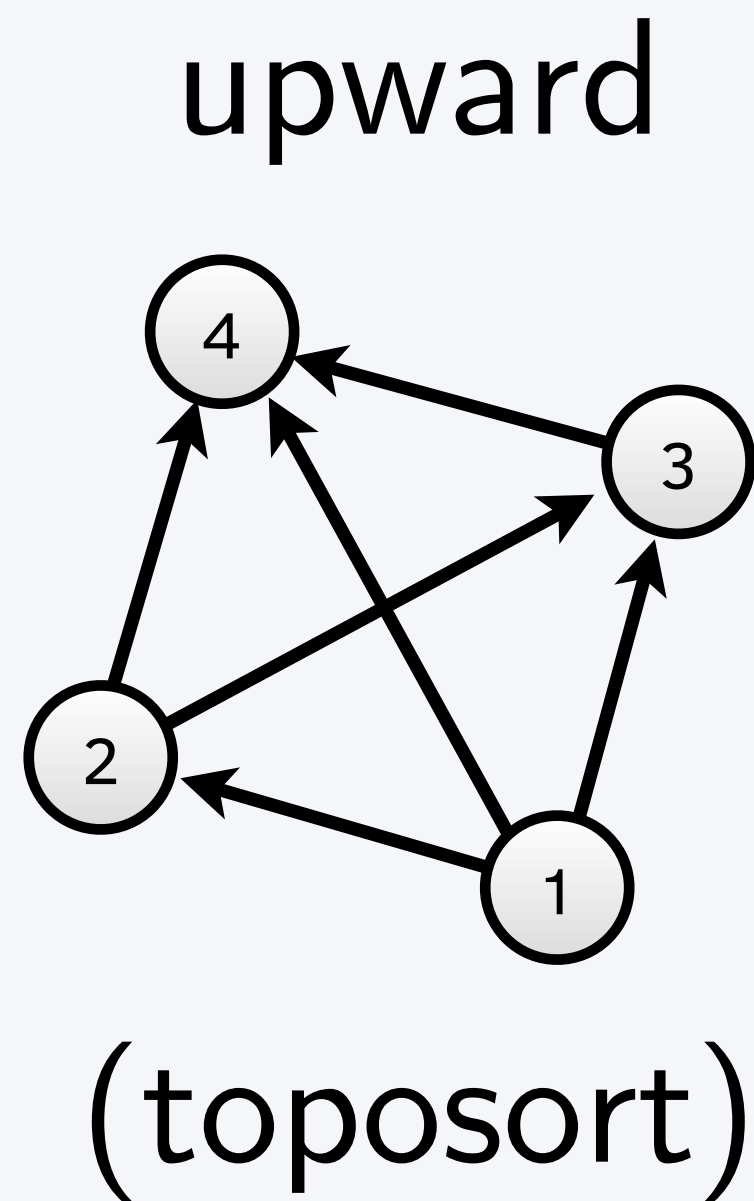
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Drawing Directed Graphs

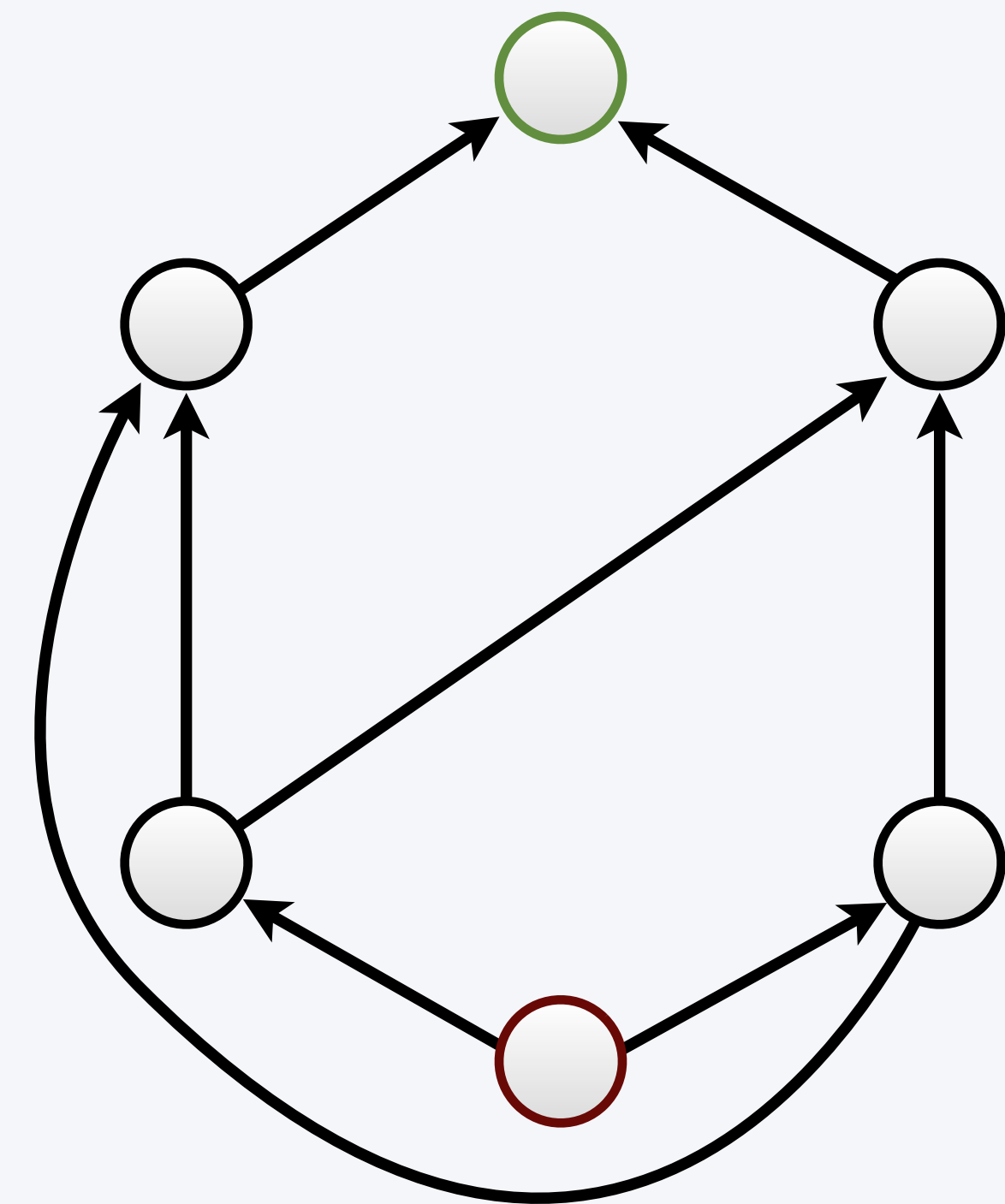
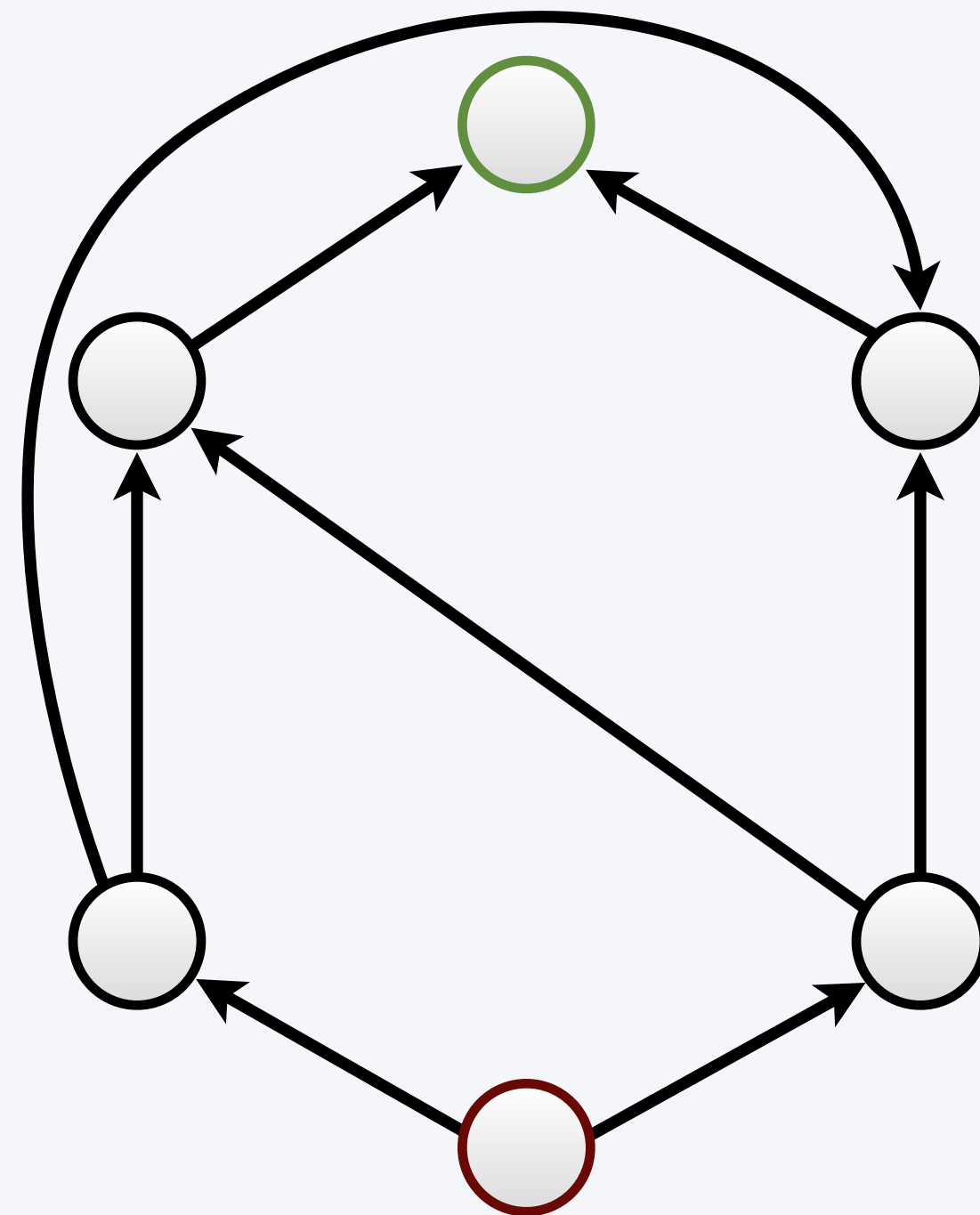
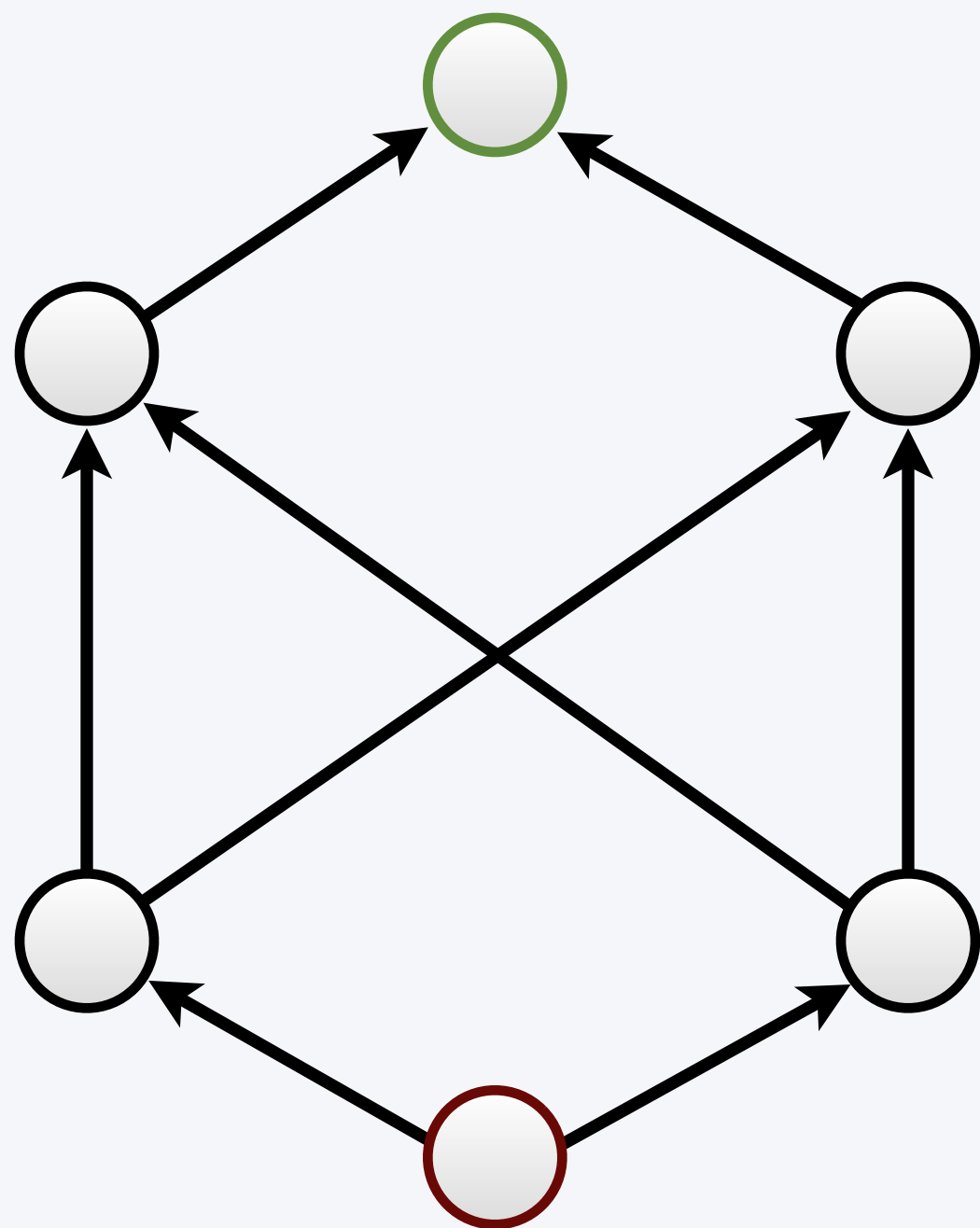
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Acyclicity is not enough



Checking Upward Planarity

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[GT95a] Ashim Garg and Roberto Tamassia. On the computational complexity of upward and rectilinear planarity testing. In *Graph drawing*, pages 286–297. Springer, 1995.

[DBT88] Giuseppe Di Battista and Roberto Tamassia. Algorithms for plane representations of acyclic digraphs. *Theoretical Computer Science*, 61(2):175–198, 1988.

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Checking Upward Planarity

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NP-hard in general [GT95a]

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Checking Upward Planarity

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but: nice characterization [DBT88],[K87]

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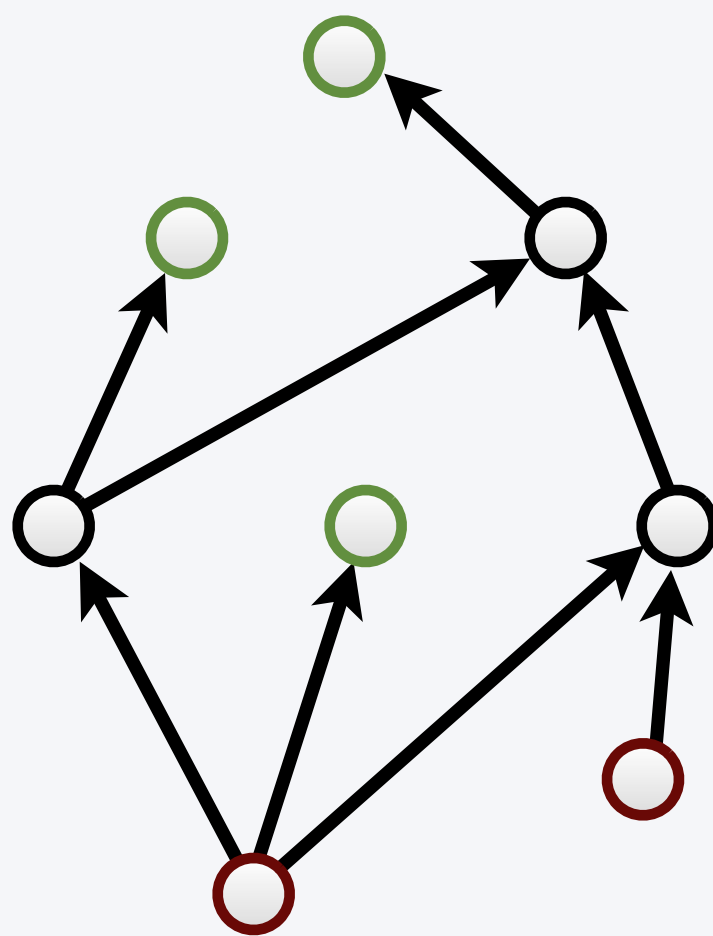
Checking Upward Planarity

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upward planar



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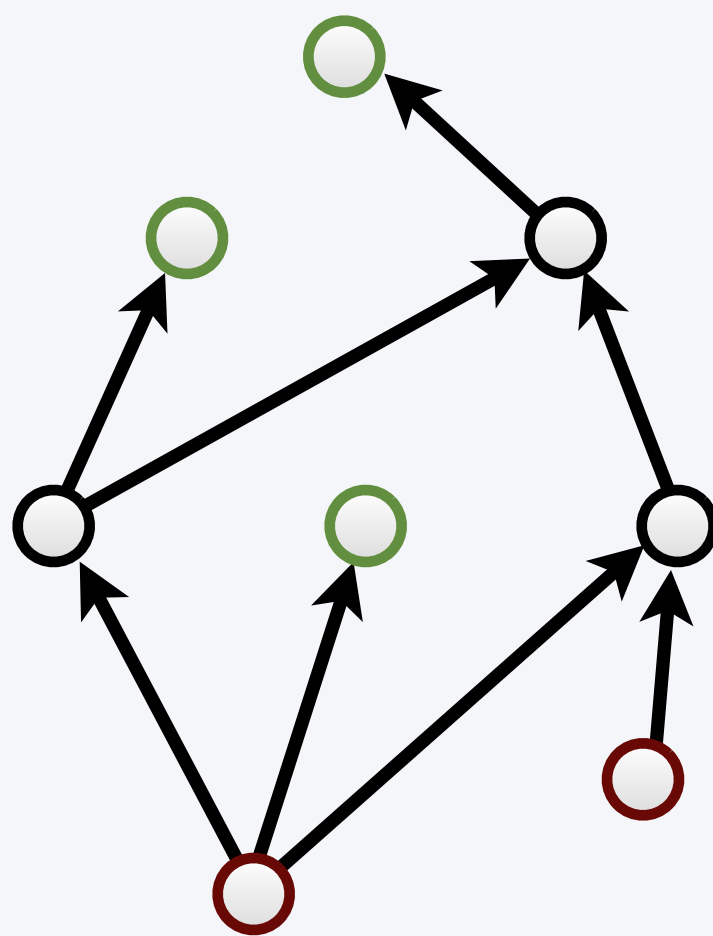
Checking Upward Planarity

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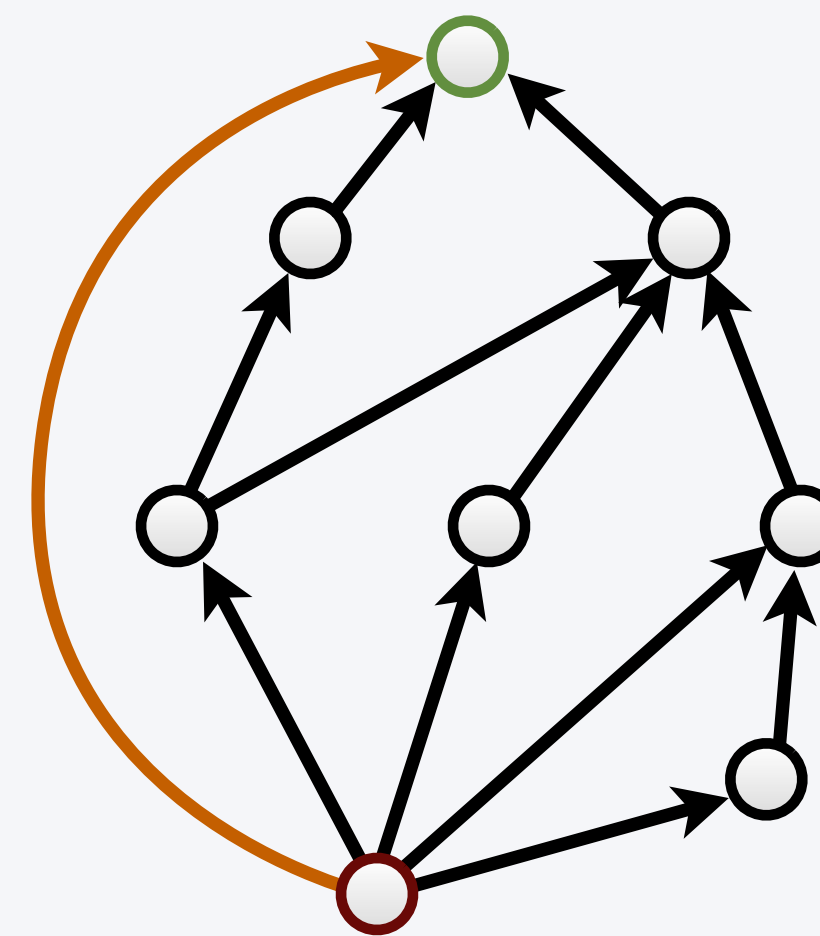
NP-hard in general [GT95a]

but: nice characterization [DBT88],[K87]

upward planar



s-t-planar



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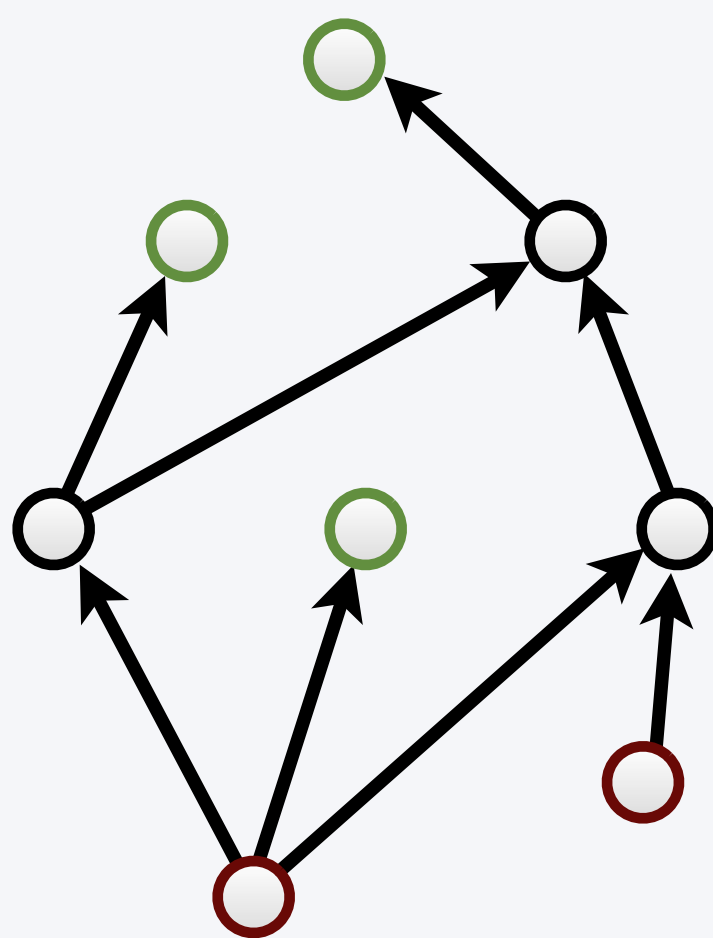
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NP-hard in general [GT95a]

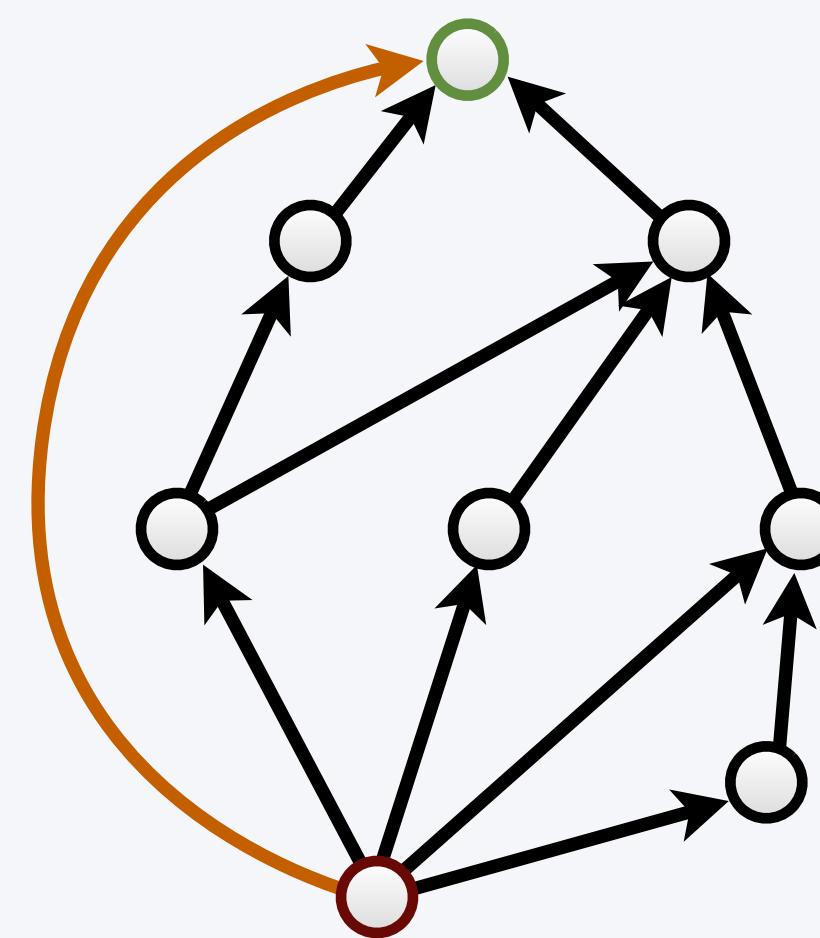
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upward planar



is spanning
subgraph of

s-t-planar



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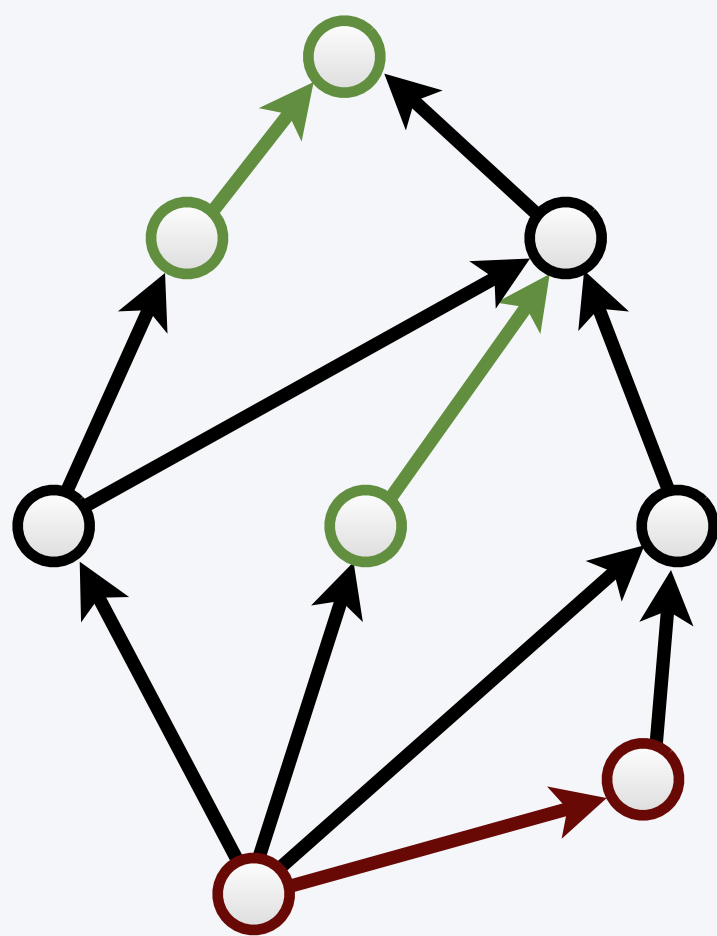
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NP-hard in general [GT95a]

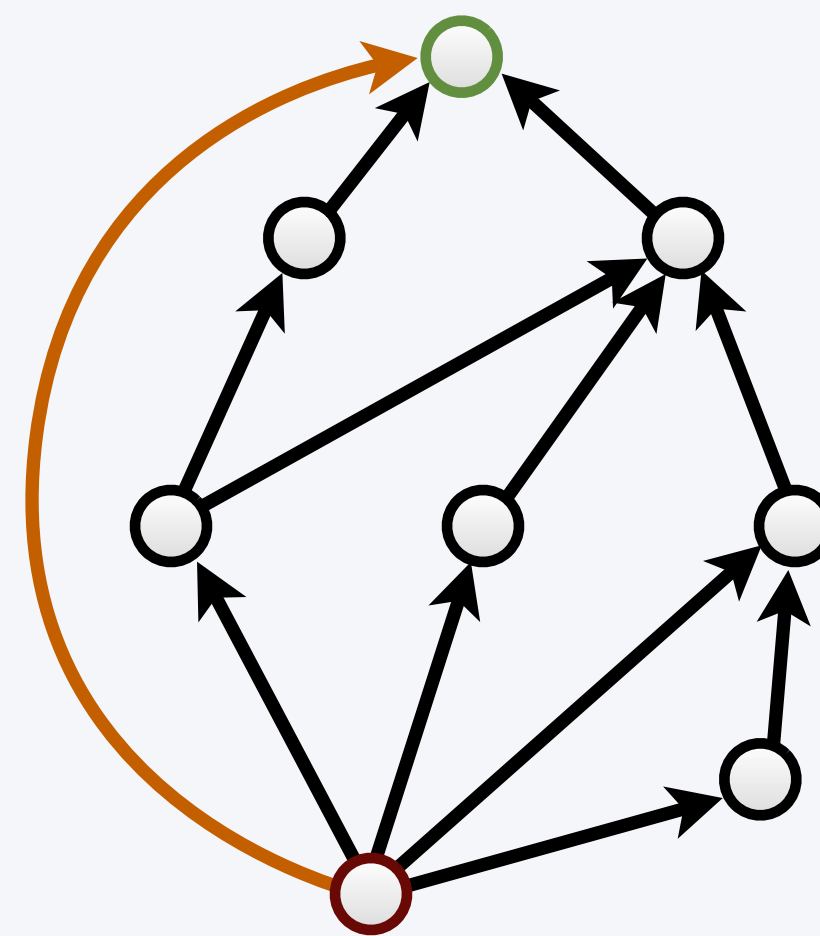
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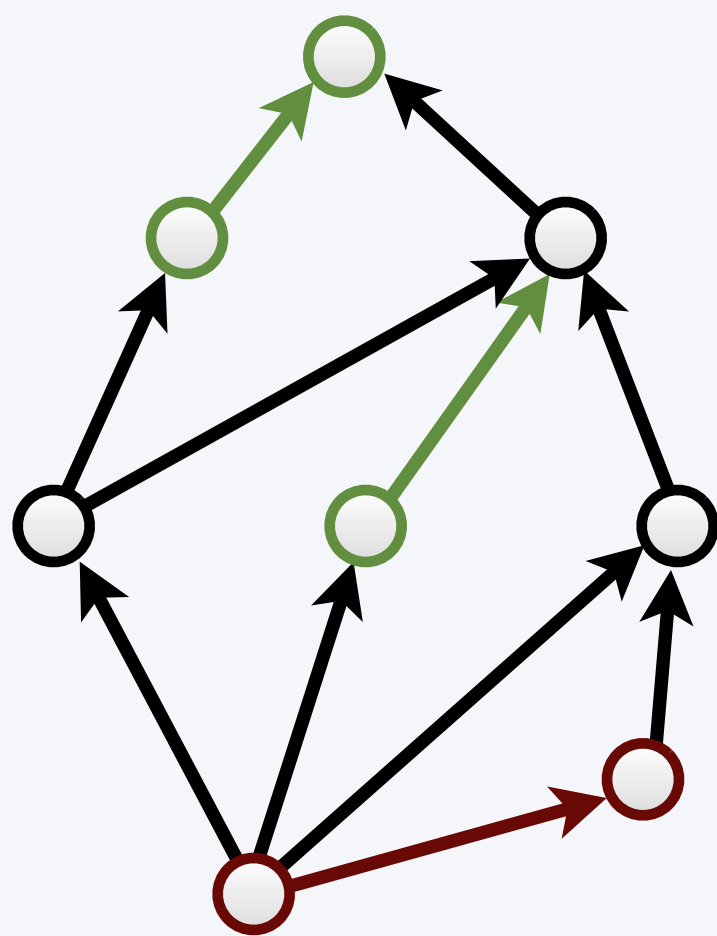
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NP-hard in general [GT95a]

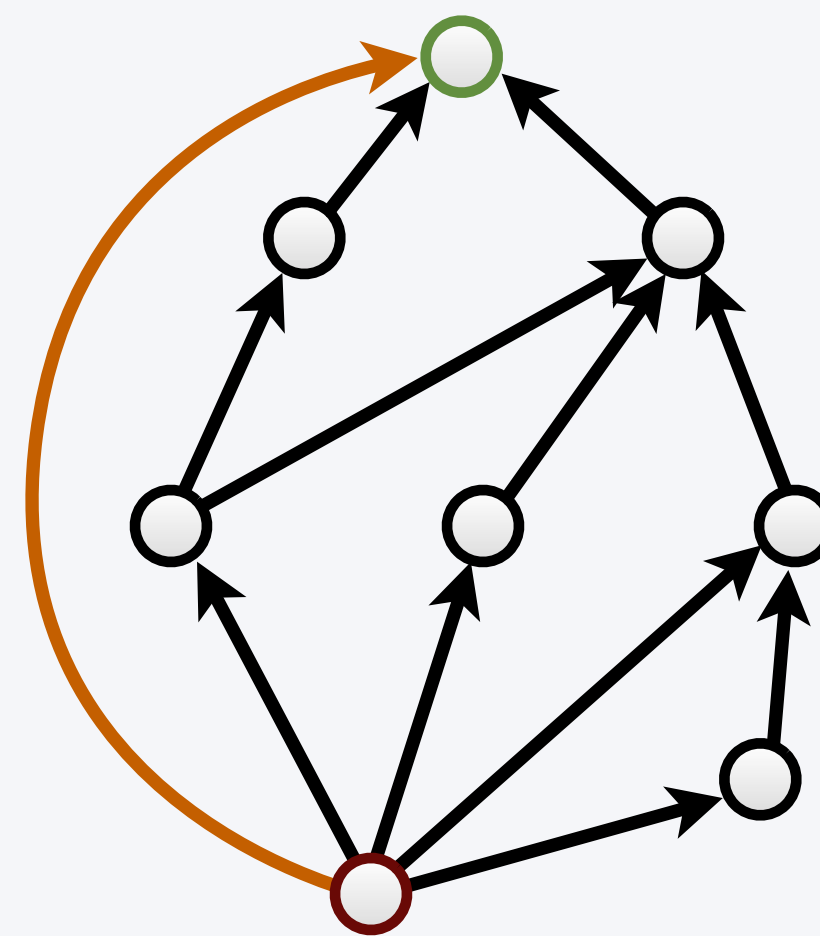
but: nice characterization [DBT88],[K87]

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\Rightarrow *NP*-complete
[GT95b]

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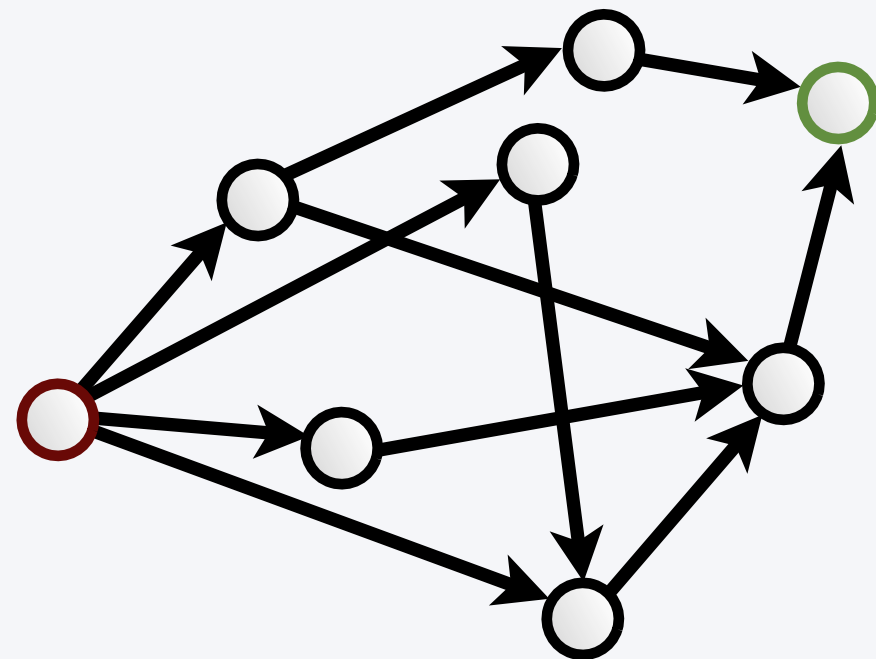
Special cases in P :

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Special cases in P :

- single source, single sink [DBT88]

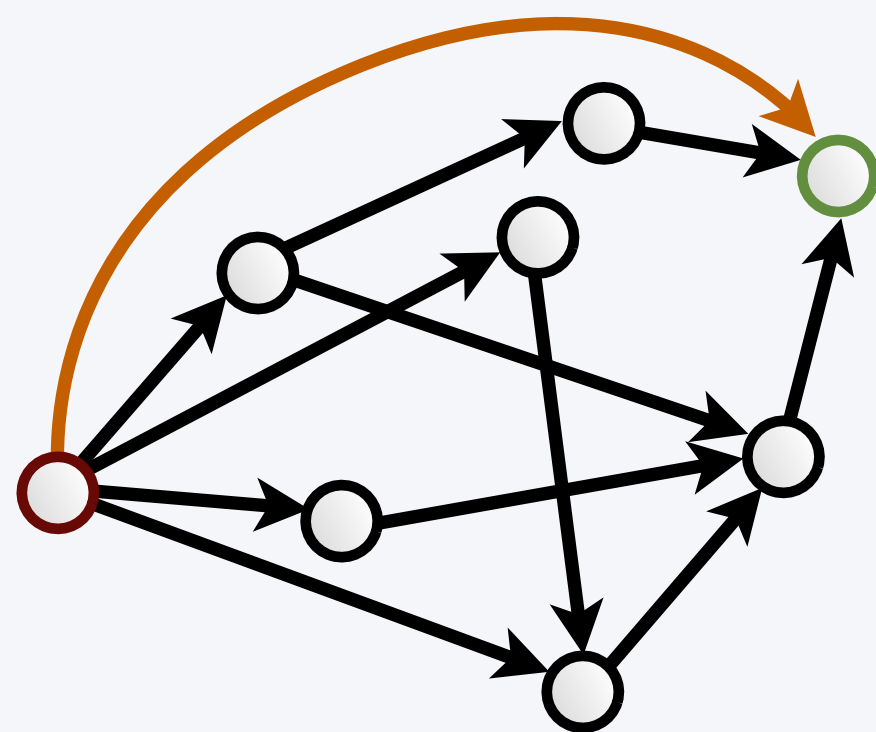


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Special cases in P :

- single source, single sink [DBT88]

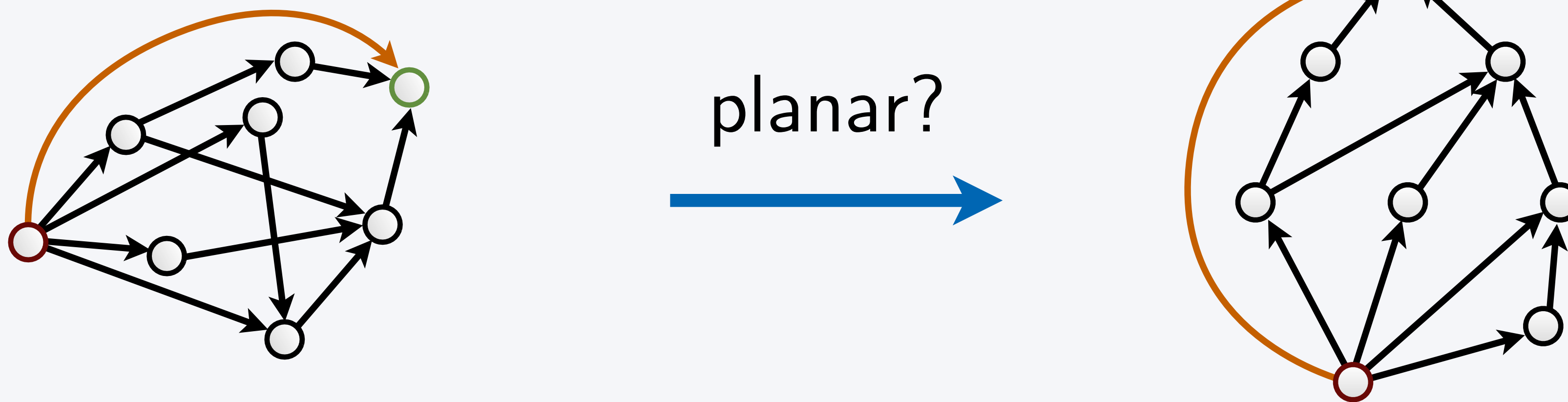


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Special cases in P :

- single source, single sink [DBT88]

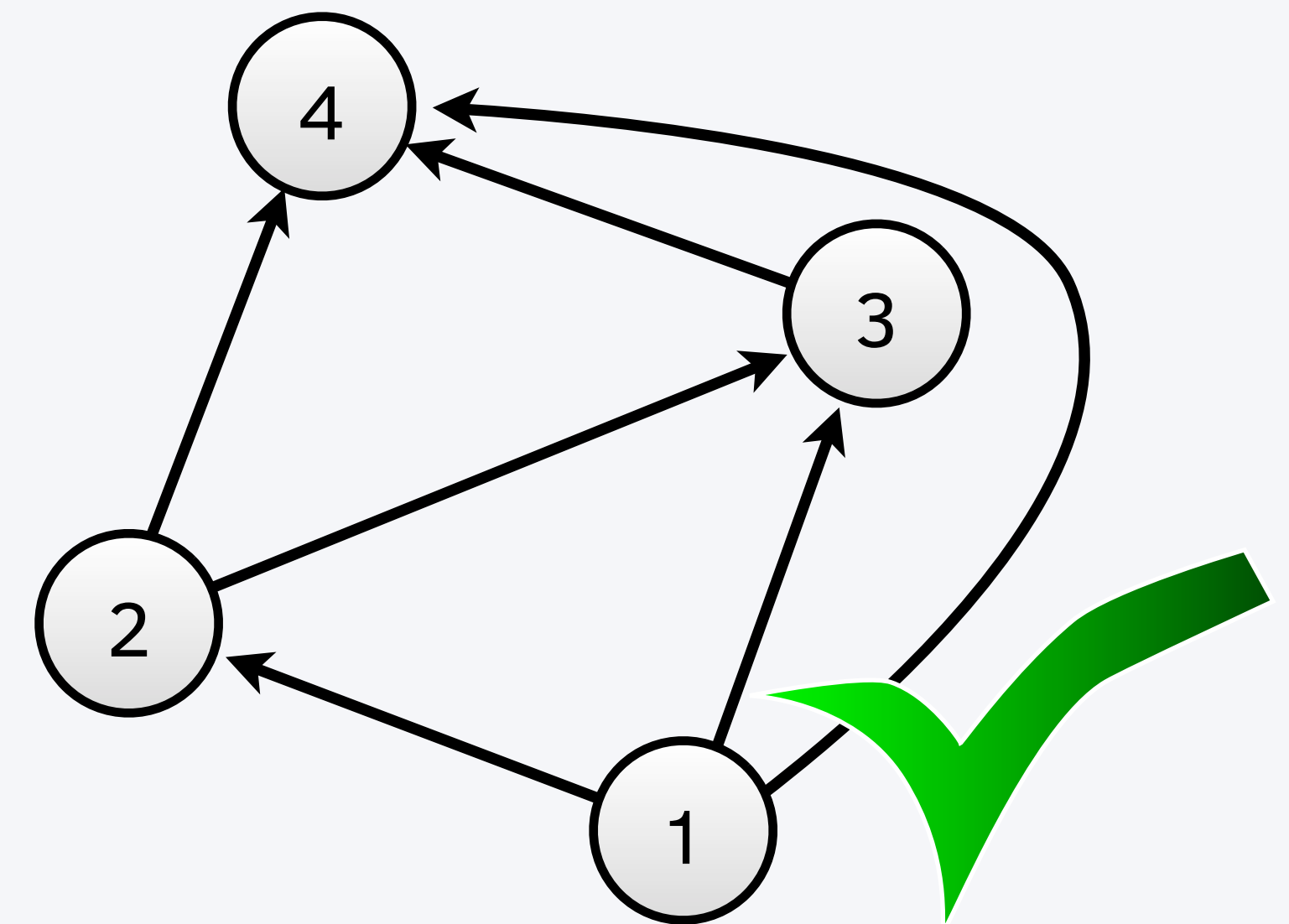
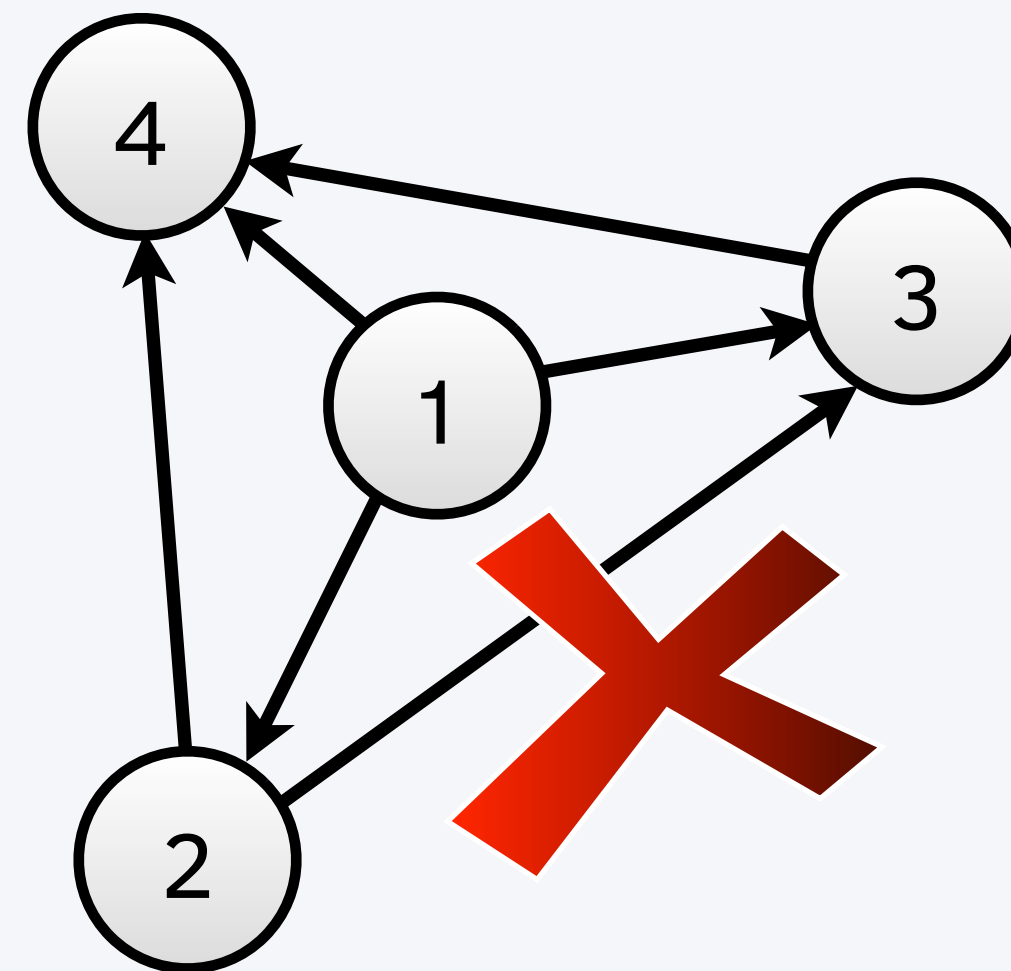


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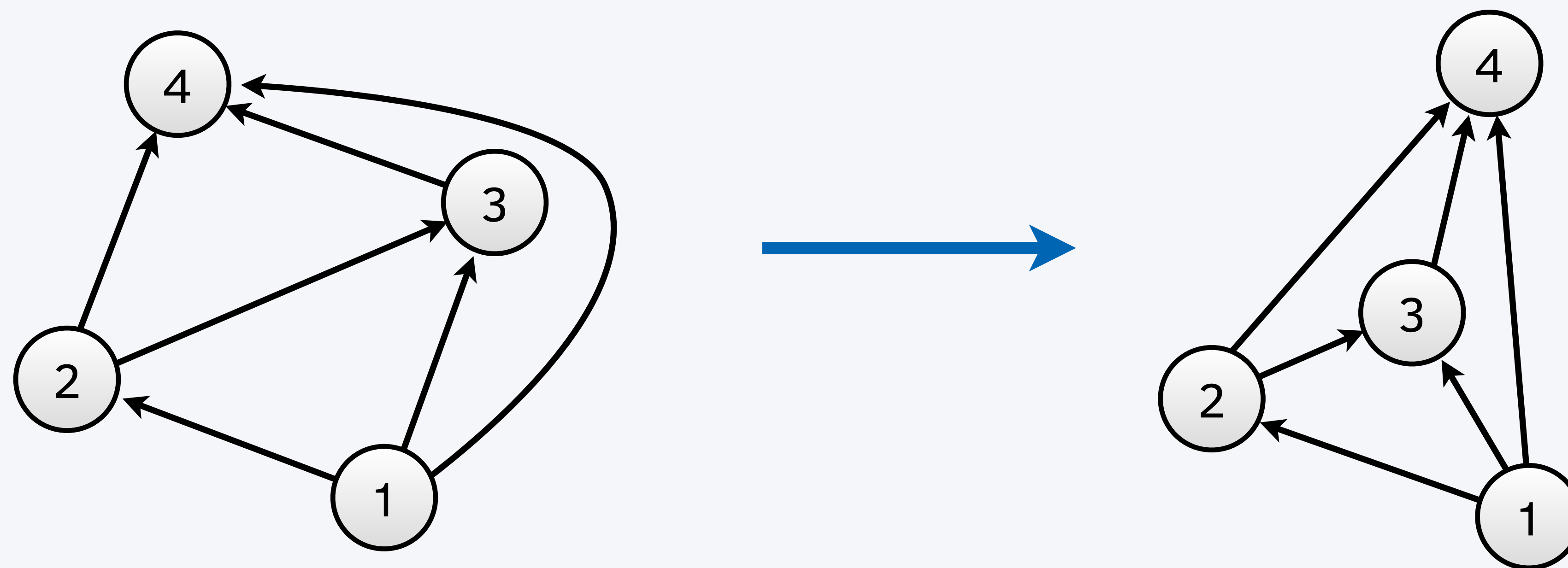
Special cases in P :

- fixed embedding [BDB91]
- ⇒ maximal planar graphs



Straight Line Upward Drawing

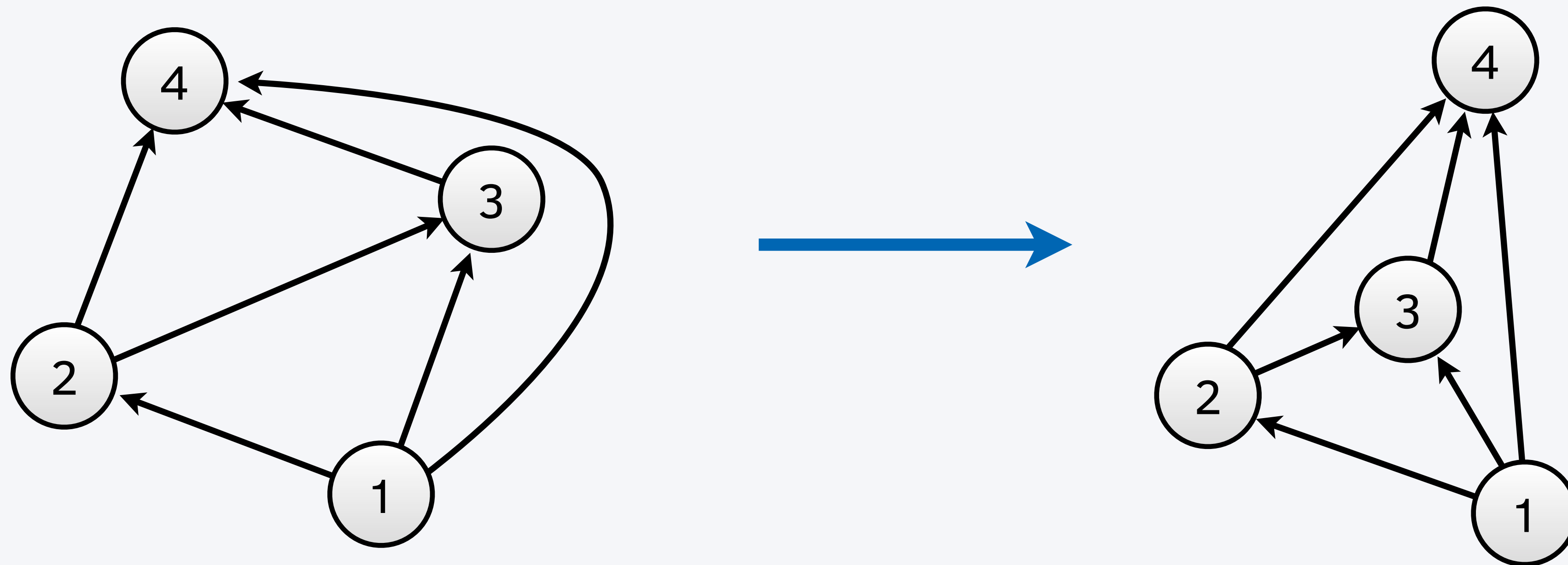
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Straight Line Upward Drawing

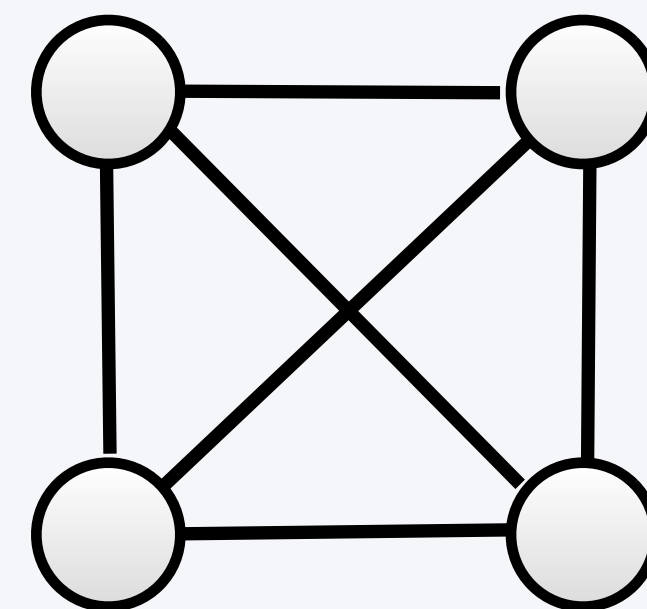
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Always possible, but might need large grid [DBTT92]



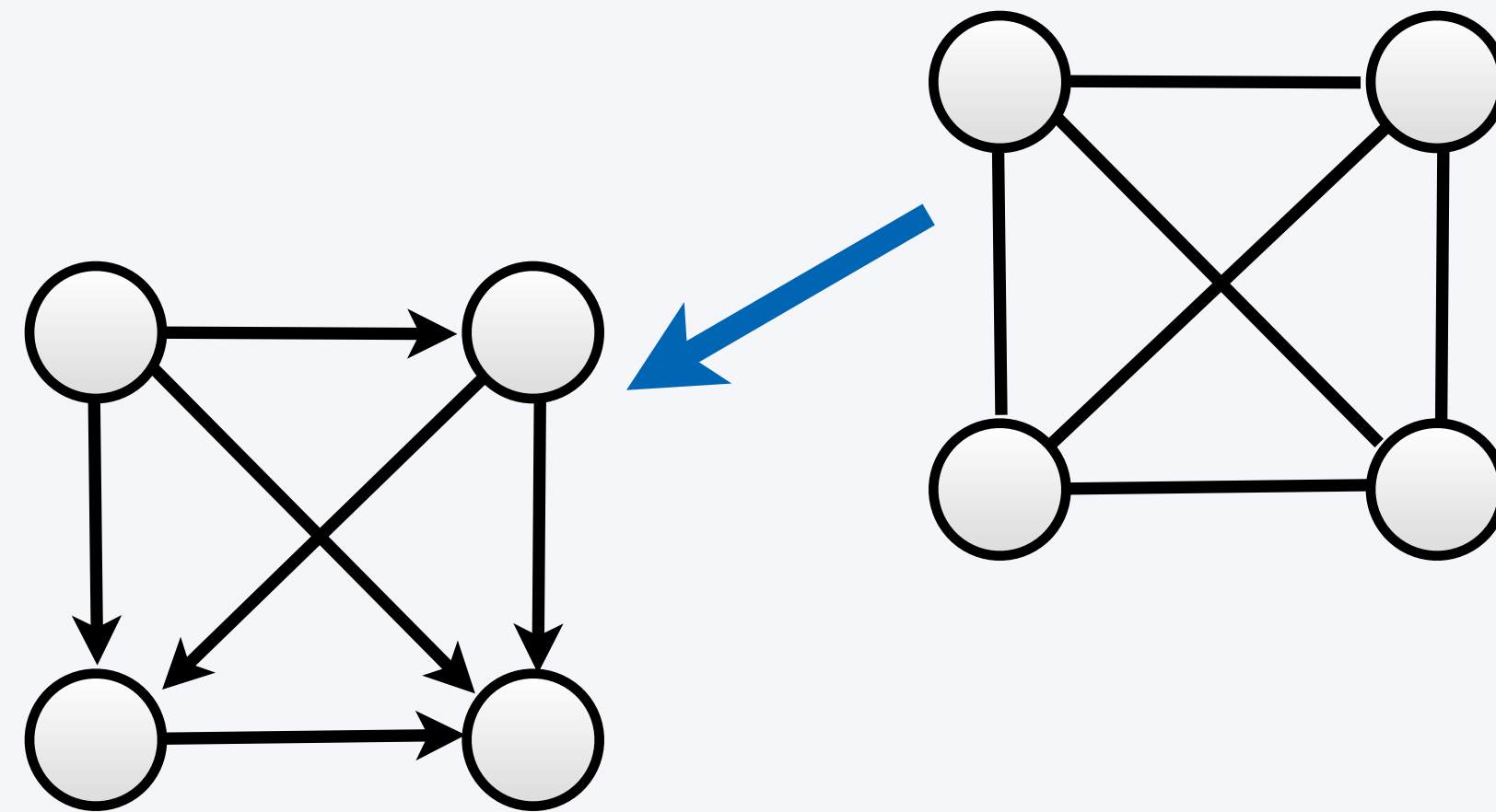
Upward Planar Orientations

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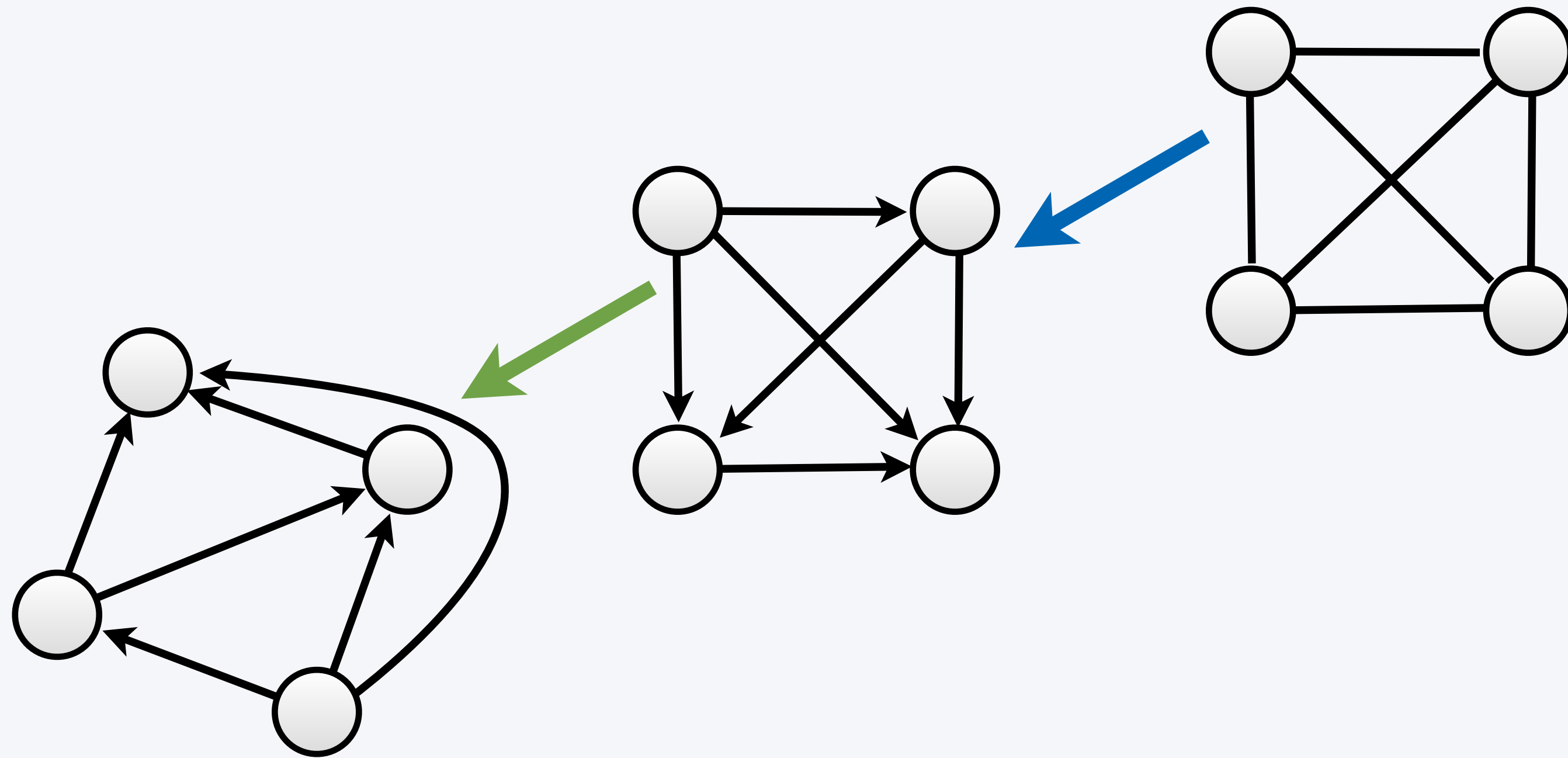
Upward Planar Orientations

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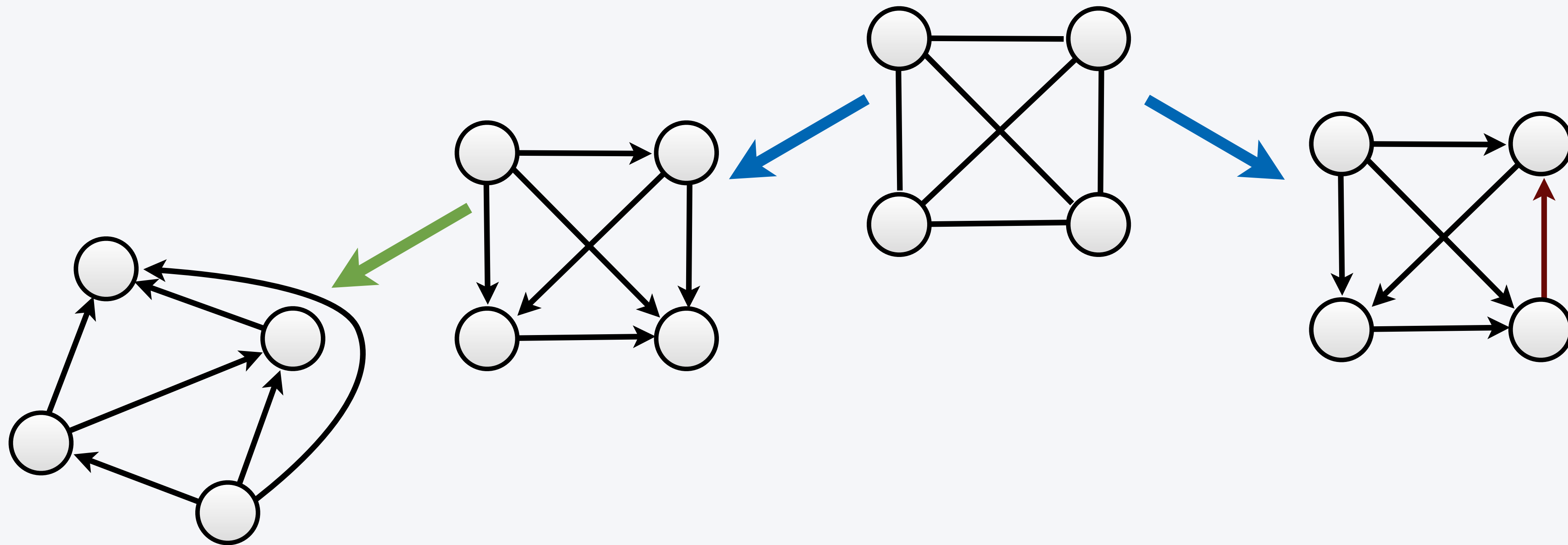
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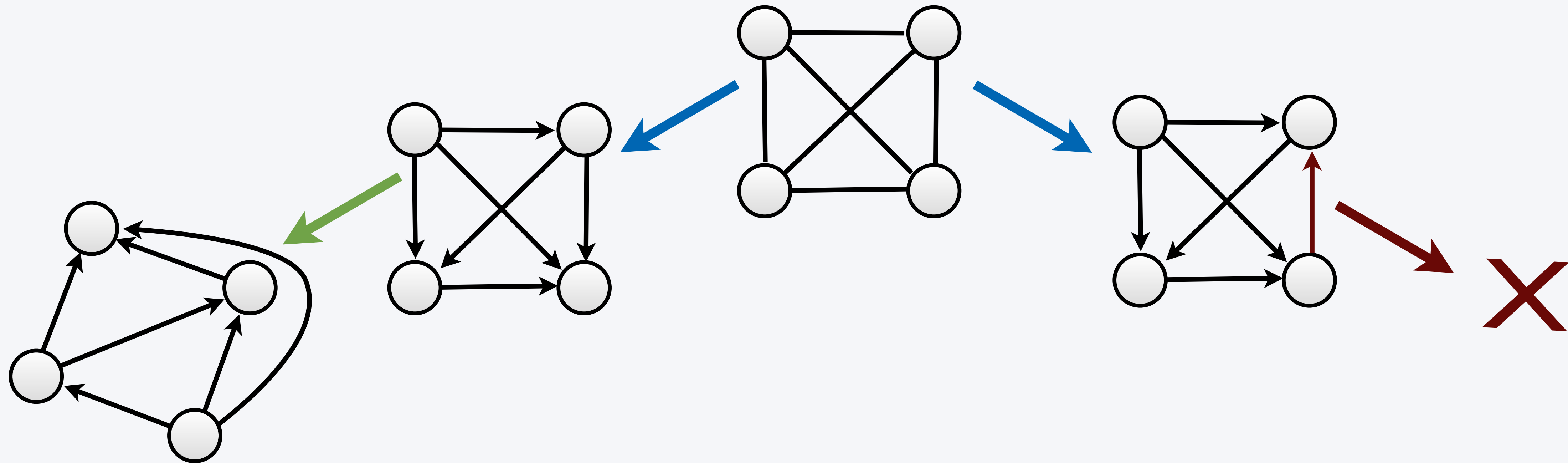
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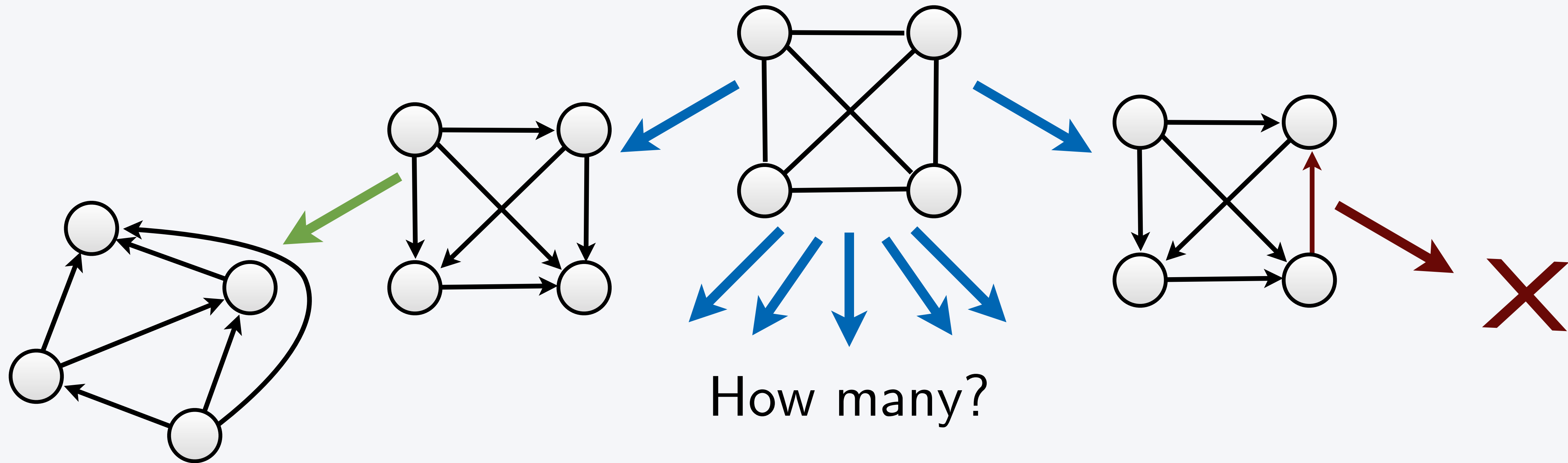
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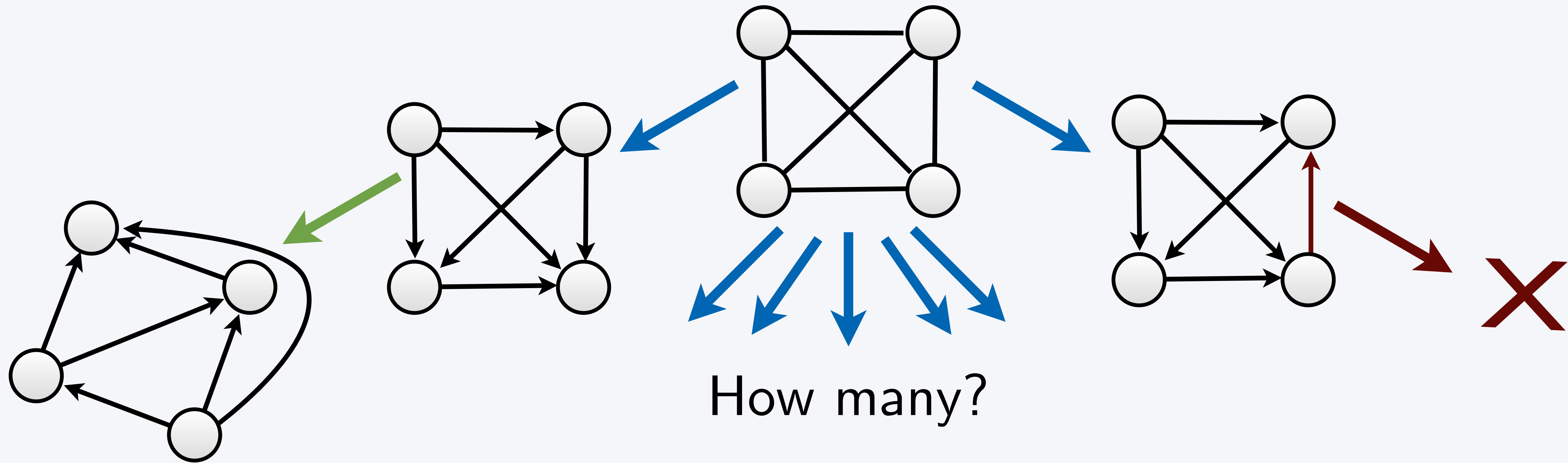
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Upward Planar Orientations

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Daniel Graf



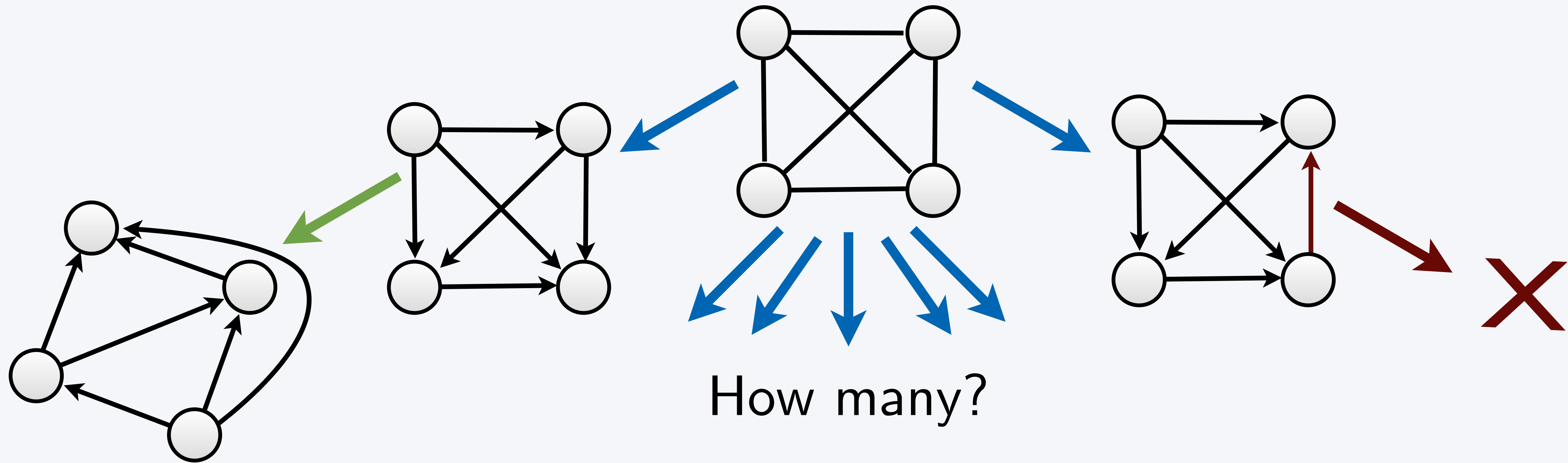
How many?

Maximal planar: some graphs with $\mathcal{O}(2^n)$ orientations

[FGW13] and some graphs with $\Omega(2.5^n)$ orientations

Upward Planar Orientations

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Maximal planar: some graphs with $\mathcal{O}(2^n)$ orientations
[FGW13] and some graphs with $\Omega(2.5^n)$ orientations

Rest: open

Checking Variations

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Daniel Graf

[BDBD98] Paola Bertolazzi, Giuseppe Di Battista, and Walter Didimo. Quasi-upward planarity. In *Graph Drawing*, pages 15–29. Springer, 1998.

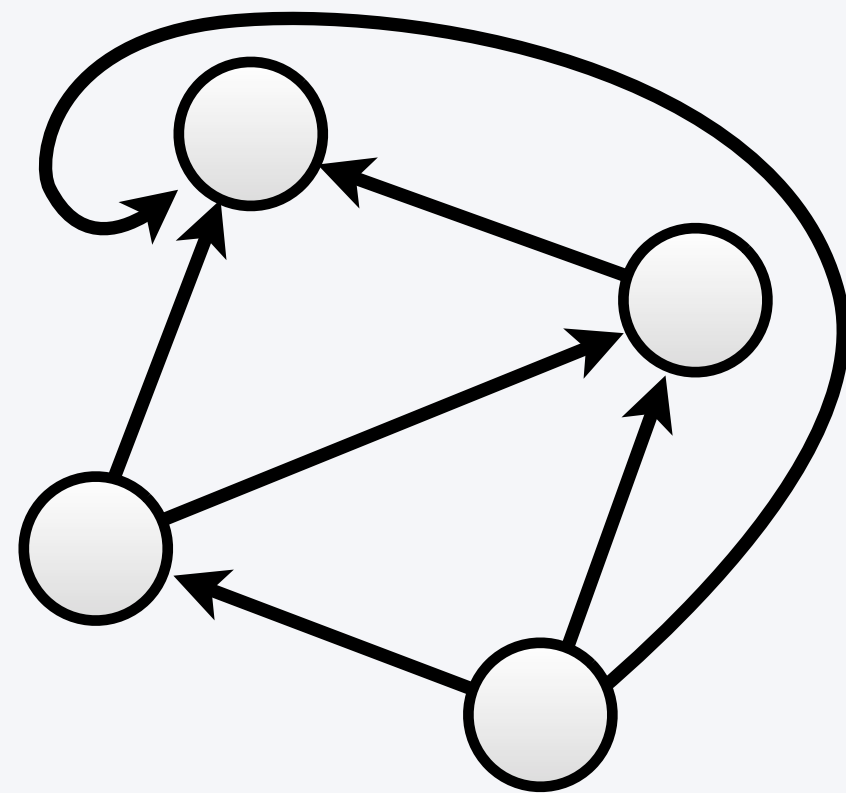
[FKPTW13] Fabrizio Frati, Michael Kaufmann, Janos Pach, Csaba D Toth, and David R Wood. On the upward planarity of mixed plane graphs. In *Graph Drawing*, pages 1–12. Springer, 2013.

[BDP14] Carla Binucci, Walter Didimo, and Maurizio Patrignani. Upward and quasi-upward planarity testing of embedded mixed graphs. *Theoretical Computer Science*, 526:75–89, 2014.

Checking Variations

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quasi upward



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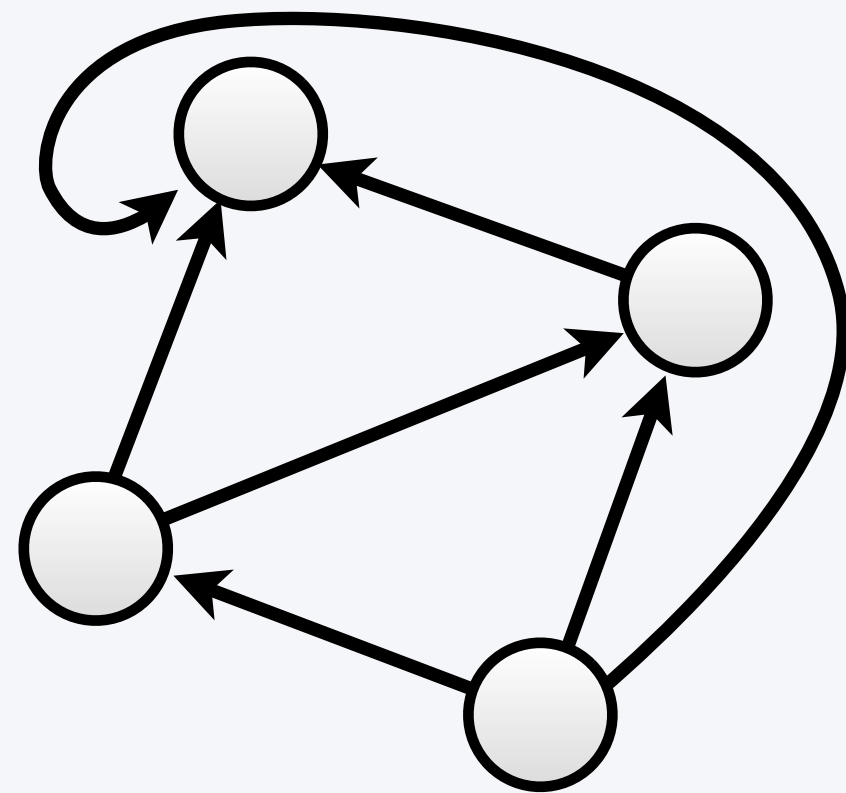
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Checking Variations

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quasi upward



[BDBD98] given ϕ , in \mathcal{P}

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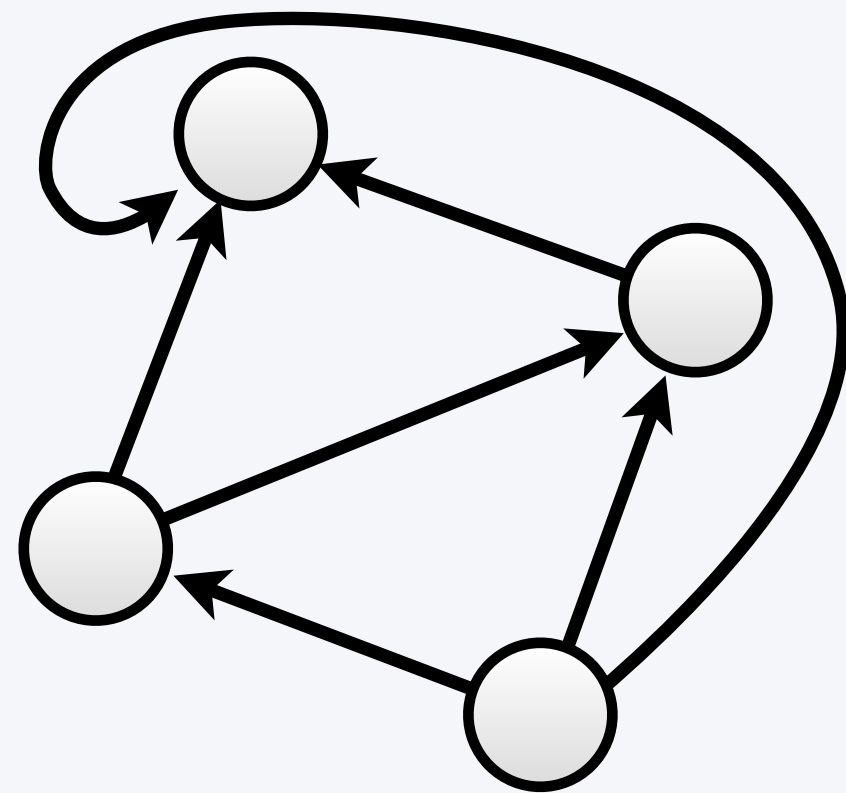
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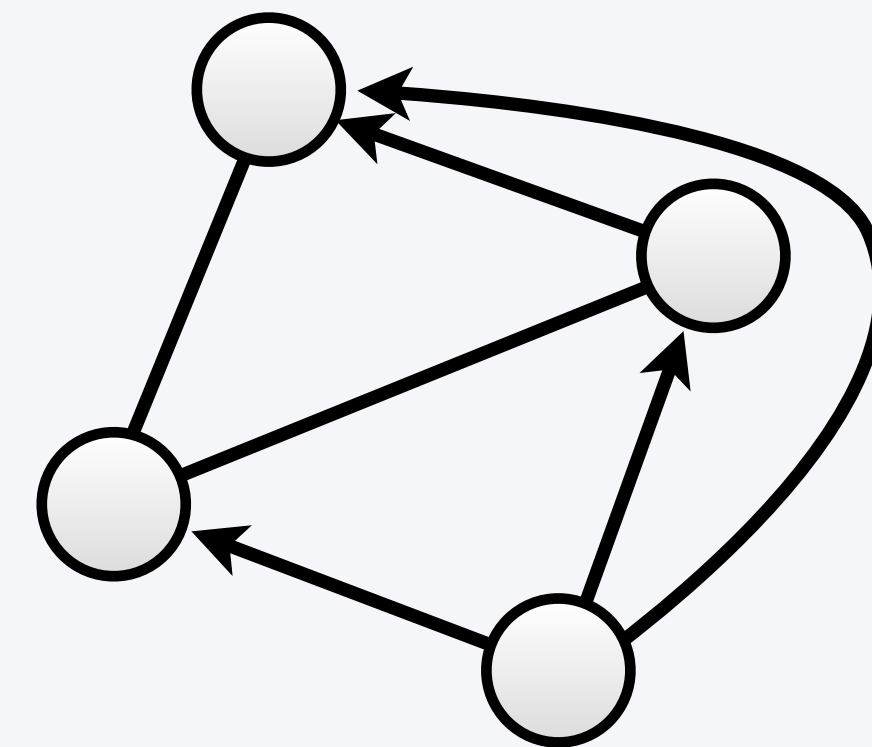
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quasi upward



[BDBD98] given ϕ , in \mathcal{P}

mixed graphs



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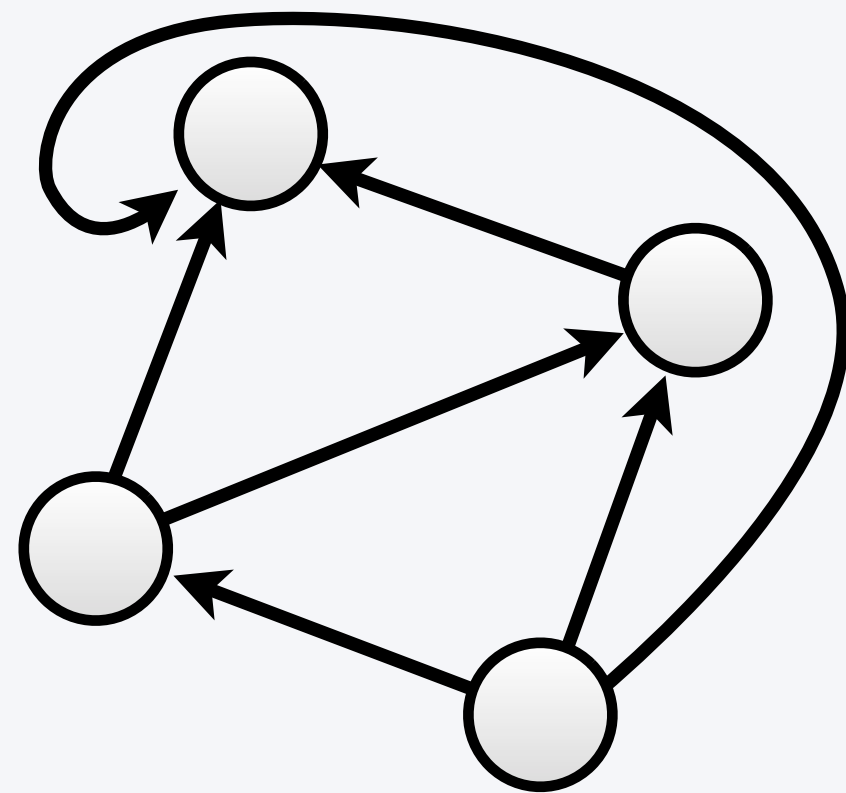
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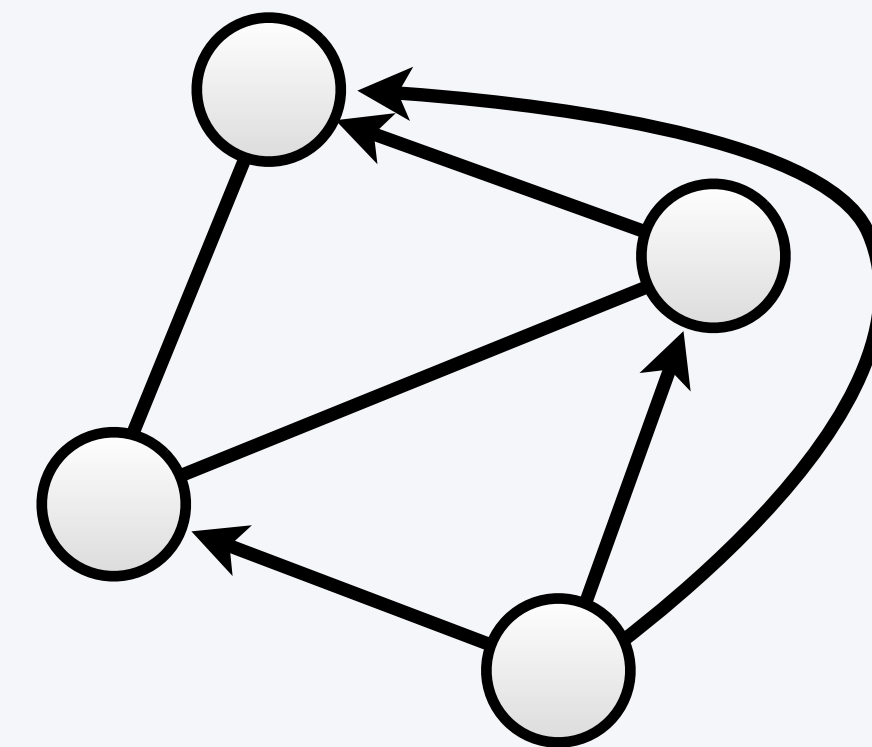
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[BDBD98] given ϕ , in \mathcal{P}

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[FKPTW13] some classes in \mathcal{P}

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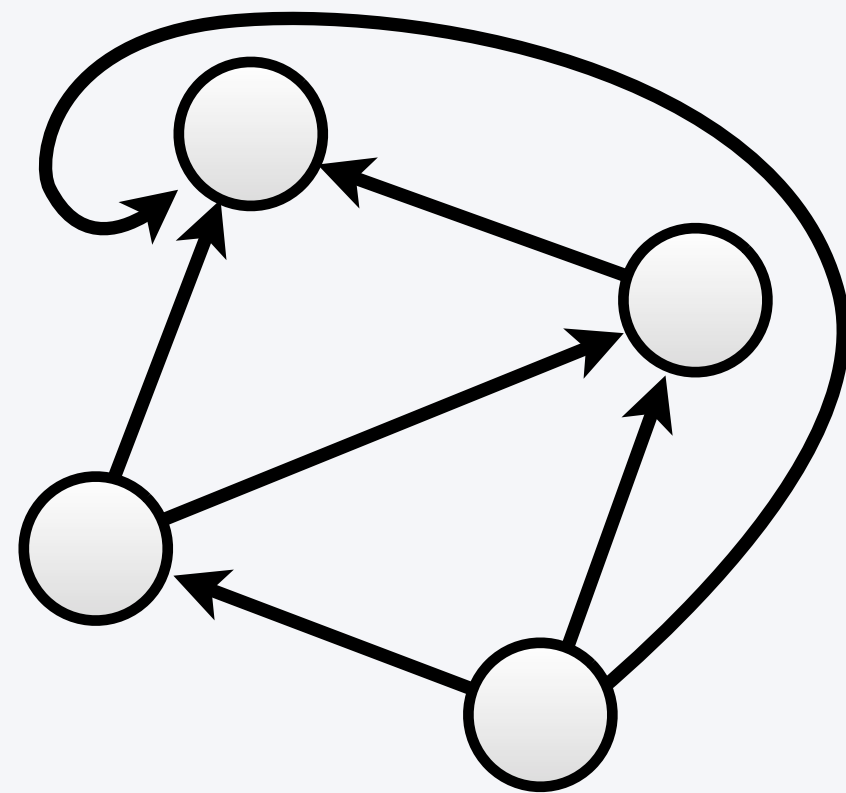
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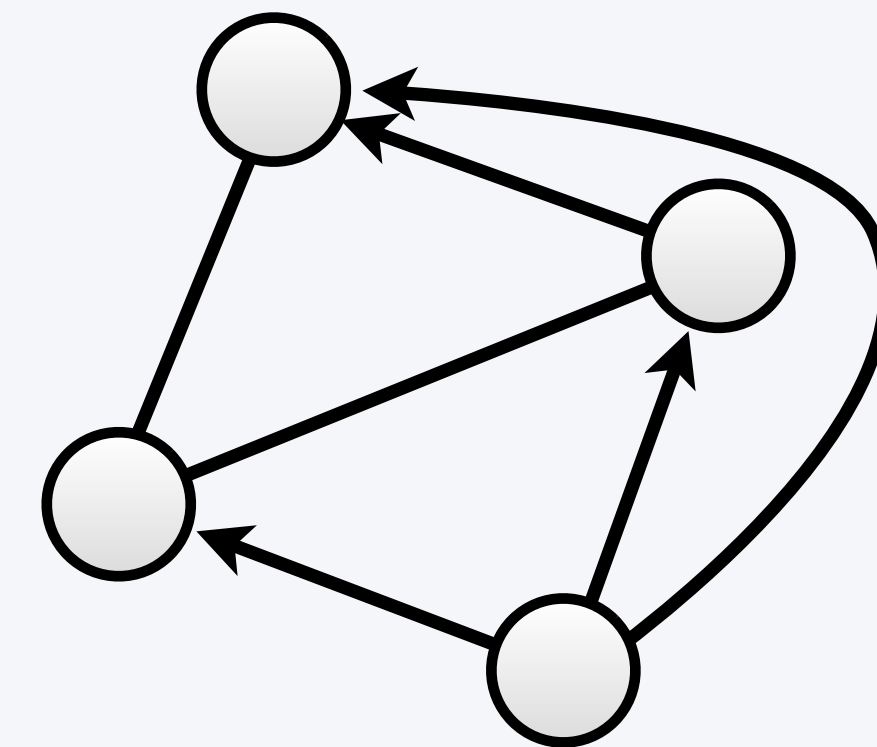
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quasi upward



[BDBD98] given ϕ , in \mathcal{P}

mixed graphs



[FKPTW13] some classes in \mathcal{P}

Open: mixed but fixed ϕ ?

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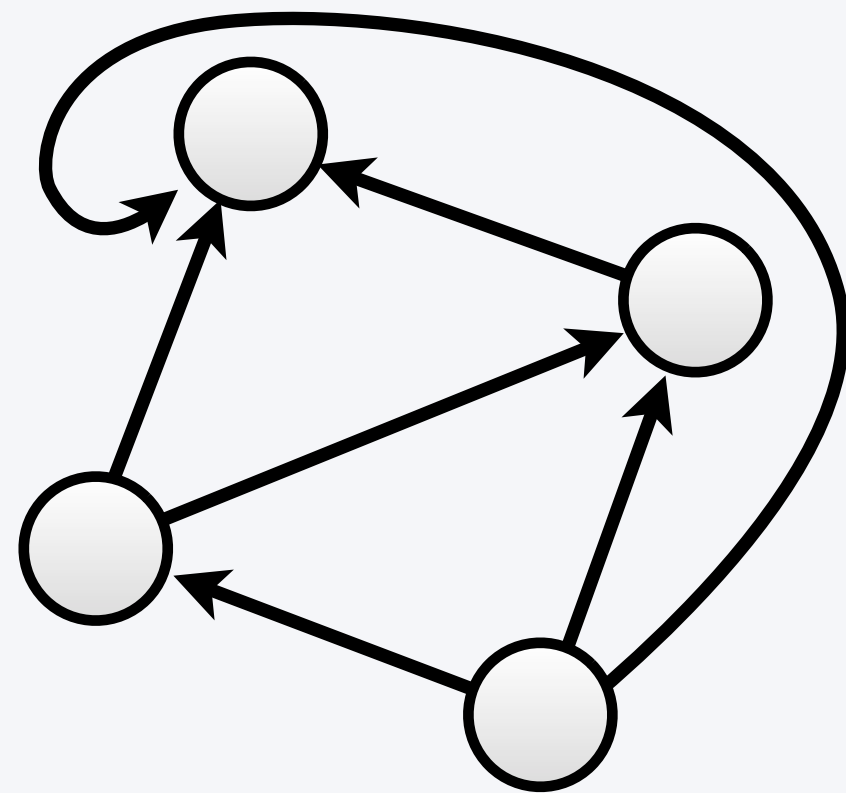
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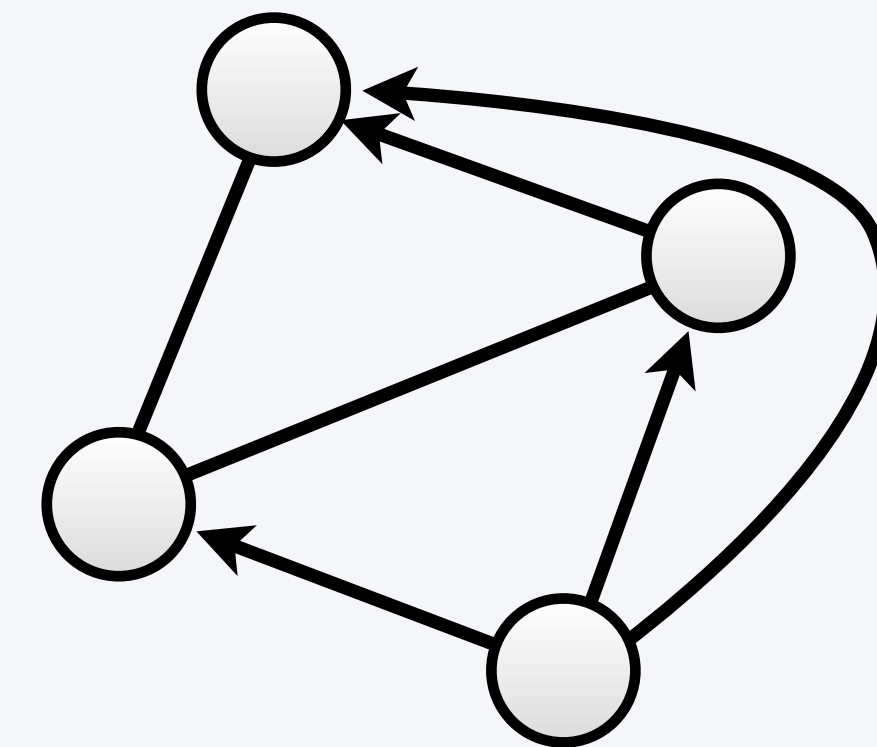
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quasi upward



[BDBD98] given ϕ , in \mathcal{P}

mixed graphs



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- mixed \rightarrow in \mathcal{P}

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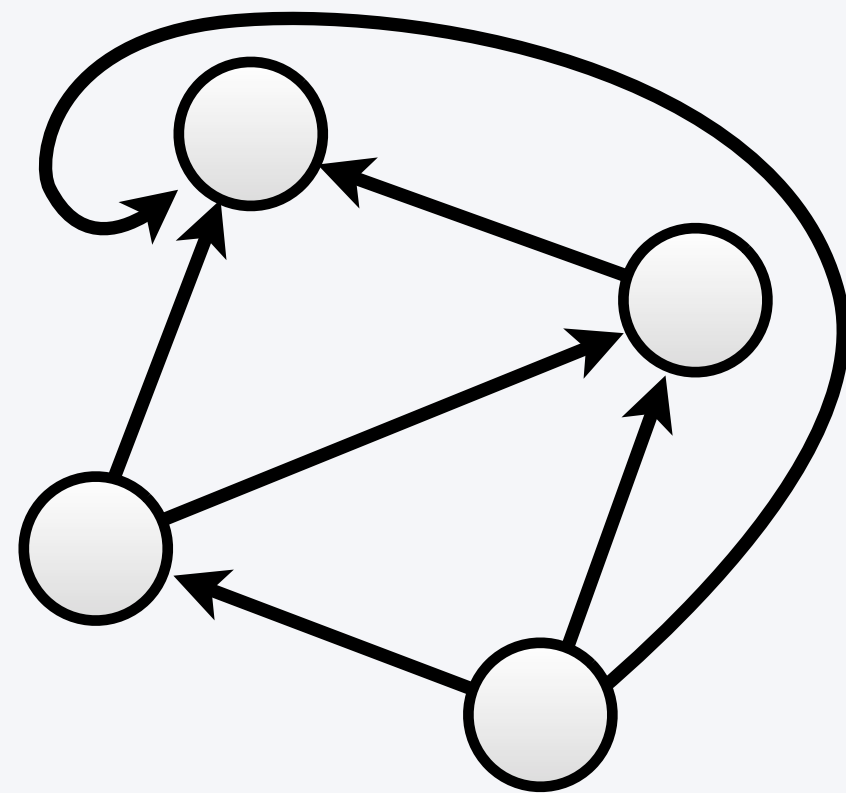
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Checking Variations

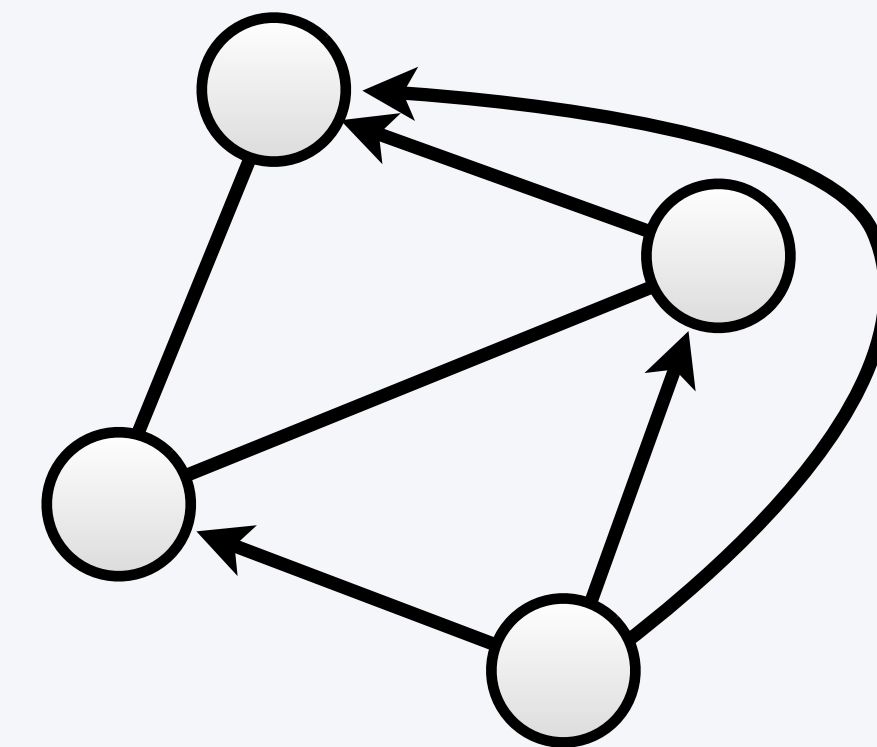
ETH Zürich
Daniel Graf

quasi upward



[BDBD98] given ϕ , in P

mixed graphs



[FKPTW13] some classes in P

Open: mixed but fixed ϕ ?

- mixed \rightarrow in P + quasi \rightarrow NP-hard [BDP14]

[BDBD98] Paola Bertolazzi, Giuseppe Di Battista, and Walter Didimo. Quasi-upward planarity. In Graph Drawing, pages 15–29. Springer, 1998.

[FKPTW13] Fabrizio Frati, Michael Kaufmann, Janos Pach, Csaba D Toth, and David R Wood. On the upward planarity of mixed plane graphs. In Graph Drawing, pages 1–12. Springer, 2013.

[BDP14] Carla Binucci, Walter Didimo, and Maurizio Patrignani. Upward and quasi-upward planarity testing of embedded mixed graphs. Theoretical Computer Science, 526:75–89, 2014.